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ACTIVE-R BANDPASS FILTER DESIGN USING HYBRID-PI TRANSISTOR MODE--ETC(U)

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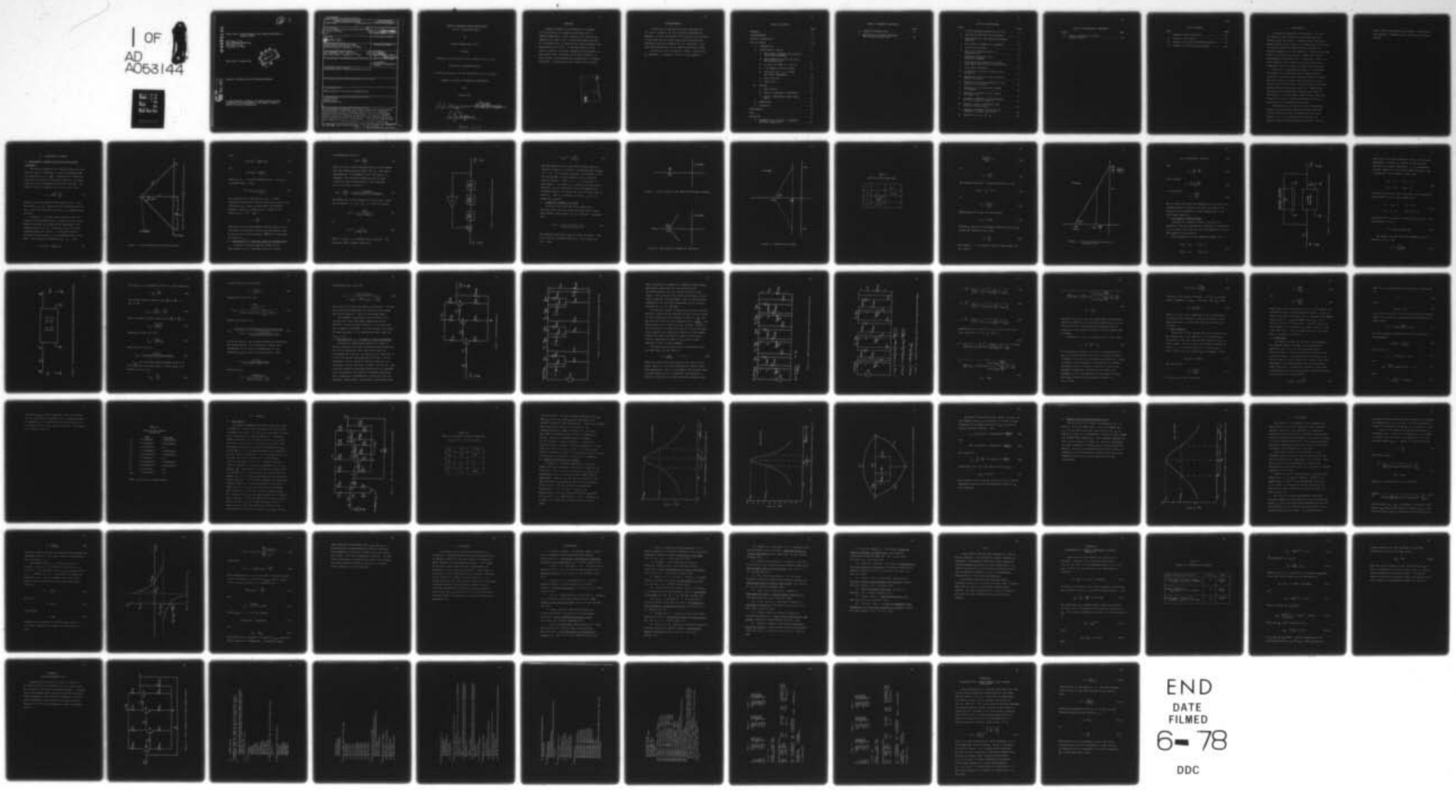

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A design procedure is developed for a bandpass filter utilizing five bipolar transistors and no external capacitors. Base resistors of a three-stage amplifier are used to set a third-order open-loop pole at a location given in terms of the center frequency and bandwidth of the filter. A feedback resistor, also given in terms of the center frequency and bandwidth, is then used to move the poles to the final location. A test circuit is constructed and compared with a computer simulation. Limitations of the circuit are discussed.

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I. INTRODUCTION

A transistor amplifier is not perfect. Its gain is limited at high frequencies by various physical effects within the device. Classical active RC filter design ignores these effects to a large extent by the use of external capacitors whose effects dominate the internal parameters of the transistor. However, since the advent of integrated circuitry, a capacitor occupies considerably larger space than a transistor. Hence, if these external design capacitors can be eliminated, a substantial savings in terms of space can be realized.

Early work by Paphitas and Murata (1) and Berman and Newcomb (5) demonstrated that bandpass filters using only transistors and resistors could be realized, but their design utilized a two- or three-transistor "stage" as a building block, and thus their realizations required excess transistors (over 11). Additionally, the Berman and Newcomb design required two types of transistors: one type whose cutoff was well above the frequency of interest and one type whose cutoff was below or near the frequency of interest.

Numerous works (2-4 and 6-16) have extended the design of capacitor-less filters using the one-pole roll-off characteristics of operational amplifiers. However, two or more operational amplifiers are required, each composed of many transistors. So once

again, excess transistors are required. The following work develops a bandpass filter using only five transistors.

II. DEVELOPMENT OF DESIGN

A. RELATIONSHIP BETWEEN POLE LOCATION AND FILTER PARAMETERS

The design parameters of a bandpass filter are the peak or "center" frequency, ω_o, and the bandwidth, BW (or alternately the $Q = \frac{\omega_o}{BW}$). Assuming that a dominant pole may be considered, the center frequency and bandwidth are easily determined by the pole location. The peak or "center" frequency is given by (19-p. 453)

$$\omega_o = \omega_m \sqrt{1 - \frac{1}{2Q^2}} \qquad (1)$$

where ω_m is the magnitude of the dominant root. Thus for high Q, $\omega_o \approx \omega_m$. Hence for pole locations close to the $j\omega$ axis (as implied by high Q), $\omega_o \approx$ imaginary part of pole.

In Figure 1, ω_L is the lower frequency where the square of the magnitude of the transfer function, $T(j\omega)$, drops to one-half the square of the magnitude of the transfer function at ω_o. Similarly, ω_H is the higher frequency where this occurs. α is the real part of the pole location, and ω_o is the imaginary part of the pole. The bandwidth is defined as $\omega_H - \omega_L$. Since

$$|T(j\omega_L)|^2 = \tfrac{1}{2}|T(j\omega_o)|^2 \qquad (2)$$

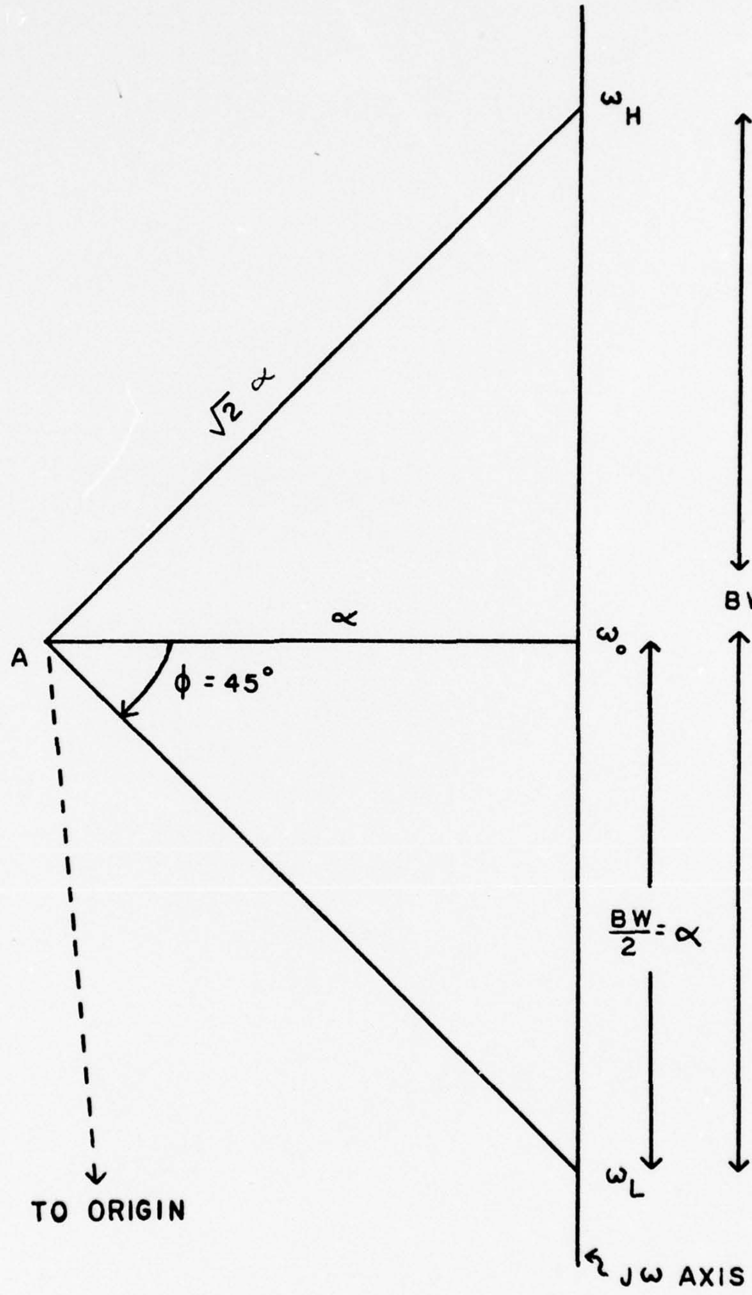


Figure 1. S-Plane Expanded Around Dominant Pole

then

$$\left|T(j\omega_L)\right| = \frac{1}{\sqrt{2}}\left|T(j\omega_o)\right| \tag{3}$$

but

$$\left|T(j\omega_o)\right| \approx \frac{1}{\alpha D_1 D_2 \ldots} \tag{4}$$

where D_1, D_2,... are the distances from $S = 0+j\omega_o$ to the other poles. Hence

$$\left|T(j\omega_L)\right| \approx \frac{1}{\sqrt{2}\ \alpha\ D_1 D_2 \ldots} \tag{5}$$

But since pole A is dominant, D_1, D_2,... remain relatively constant as S varies along the $j\omega$ axis in the vicinity of ω_o. Thus ω_L occurs where the distance between S and pole A increases by a factor of $\sqrt{2}$, which is at $\phi = 45^o$. Hence

$$\alpha = \frac{BW}{2} \tag{6}$$

Therefore, the desired bandpass characteristics can be achieved by setting the real part of the dominant pole of the transfer function equal to $\frac{BW}{2}$ and the imaginary part equal to ω_o.

B. RELATIONSHIPS OF OPEN-LOOP POLES AND FEEDBACK GAIN

A single transistor amplifier stage may be approximated with a single-pole roll-off transfer

characteristic given by

$$A(s) = \frac{A_o \omega_C}{S+\omega_C} \tag{7}$$

where ω_C is the cutoff frequency and A_o is the midband gain for frequencies much lower than ω_C. Using this approximation, the feedback circuit in Figure 2 is examined. The voltage transfer function of such a circuit is covered extensively in the literature (e.g., 23), and is given by

$$A_V(s) = \frac{V_{out}}{V_{in}} = \frac{A\omega_A B\omega_B C\omega_C}{(S+\omega_A)(S+\omega_B)(S+\omega_C)-KA\omega_A B\omega_B C\omega_C} \tag{8}$$

Now assume the cutoff frequencies for all three stages are set equal; i.e., $\omega_A = \omega_B = \omega_C$. Then (8) becomes

$$A_V(s) = \frac{ABC\omega_C{}^3}{(S+\omega_C)^3-KABC\omega_C{}^3} \tag{9}$$

or

$$A_V(s) = \frac{G}{(S+\omega_C)^3-KG} \tag{10}$$

where $G = ABC\omega_C{}^3$ is a constant and is negative. The open-loop (K=0) transfer function is

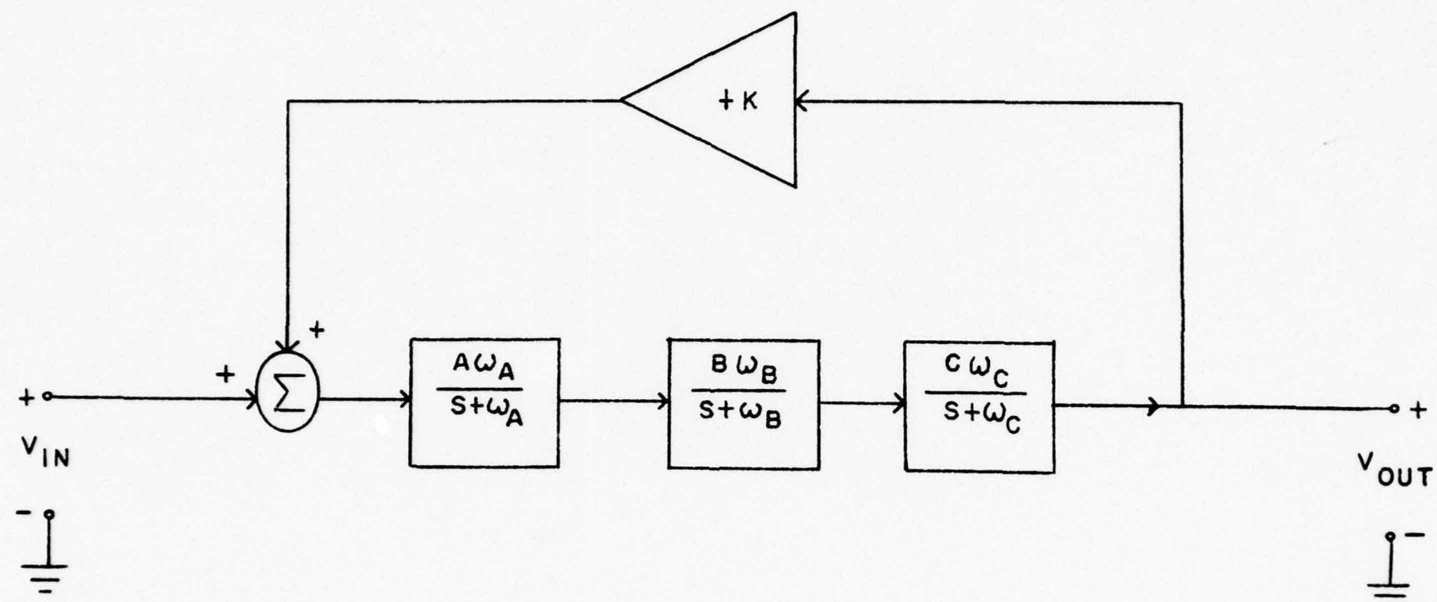


Figure 2. Three-Stage Amplifier With Feedback

$$A_{V_{OL}}(s) = \frac{G}{(S+\omega_C)^3} \tag{11}$$

The pole diagram for the open-loop transfer function appears in Figure 3, in which a third-order pole appears at $S = -\omega_C$. As feedback is applied (increasing K), the root locus departs the third-order pole in straight lines at angles $\pi - \frac{2\pi n}{3}$, $n = 0, 1, 2$ (23-chapter 7). See Figure 4. The desired ω_o and BW may be achieved by setting ω_C to the appropriate value and applying the necessary feedback to achieve the pole location shown in Figure 5. Thus it is necessary to know ω_C and K in terms of ω_o and BW.

C. <u>ω_C AND K IN TERMS OF ω_o AND BW</u>

The point where the root locus crosses the imaginary axis into the right-half-plane can be found using Routh's discriminant (23, pp. 153-154). Expanding (10),

$$A_V(s) = \frac{G}{S^3+3\omega_C S^2+3\omega_C{}^2 S+\omega_C{}^3-KG} \tag{12}$$

The Routhian coefficient table is given in Table I. The roots are pure imaginary when the S^1 row is equal to zero. Thus

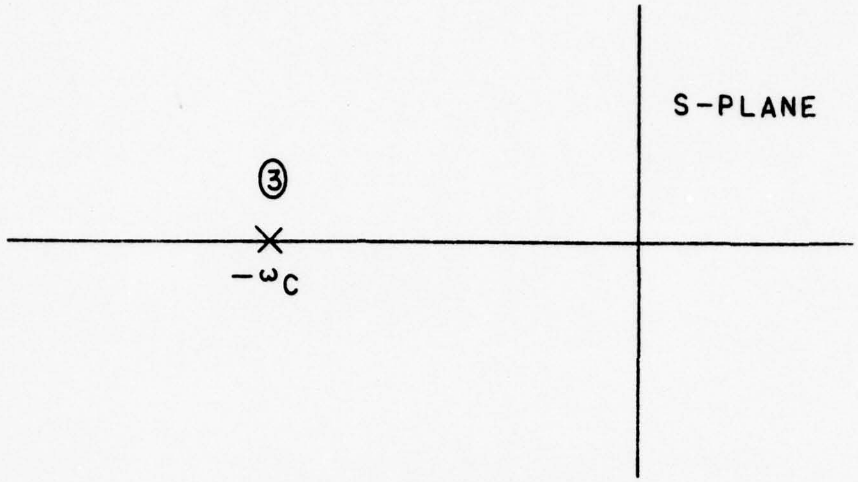


Figure 3. Pole Diagram For The Open-Loop Transfer Function

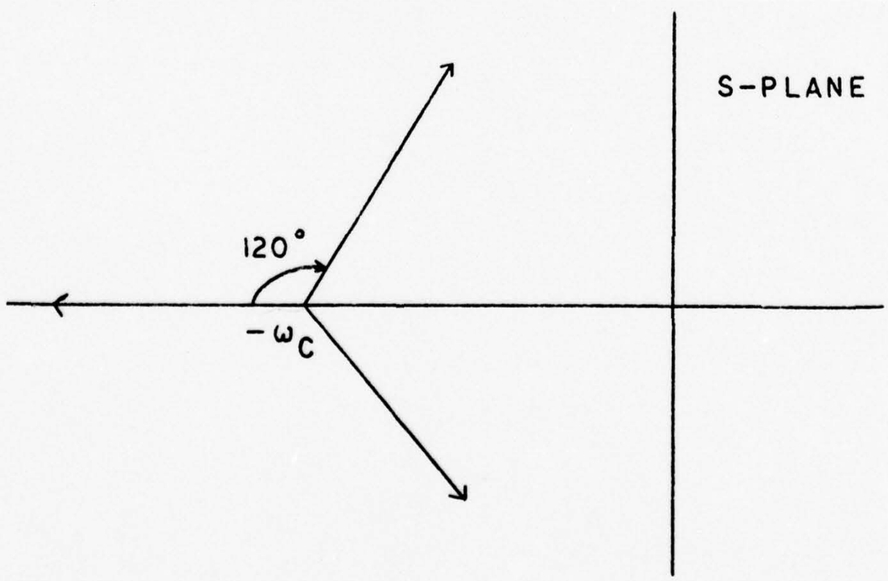


Figure 4. Root Locus As Feedback Is Increased

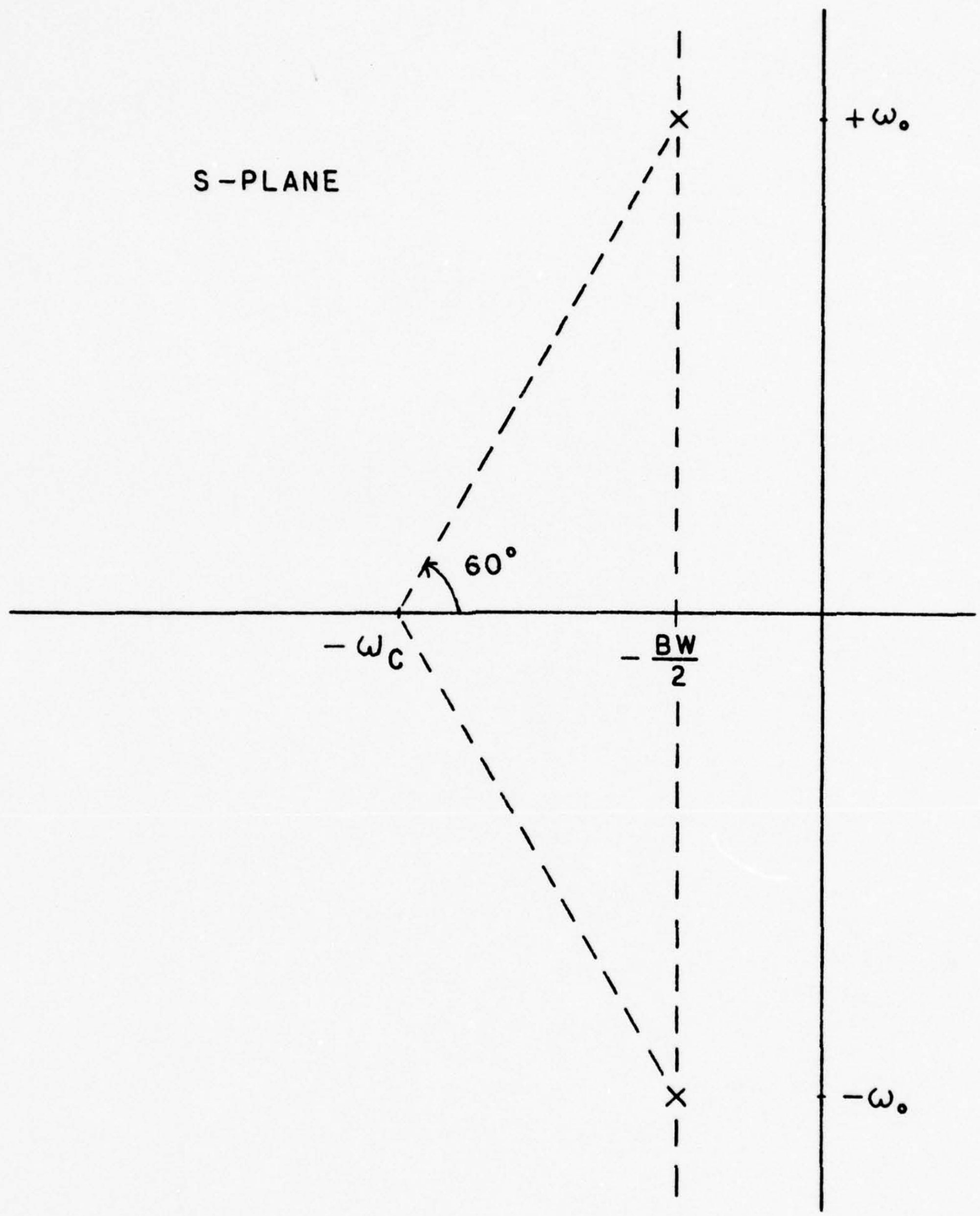


Figure 5. Desired Pole Diagram

TABLE I

ROUTHIAN COEFFICIENT ARRAY

S^3	1	$3\omega_C{}^2$
S^2	$3\omega_C$	$\omega_C{}^3-KG$
S^1	$\frac{8\omega_C{}^3+KG}{3\omega_C}$	
S^0	$\omega_C{}^3-KG$	

$$\frac{8\omega_C{}^3 + KG}{3\omega_C} = 0 \tag{13}$$

or

$$K = \frac{-8\omega_C{}^3}{G} \tag{14}$$

The auxiliary equation is obtained from the S^2 row:

$$3\omega_C S^2 + \omega_C{}^3 - KG = 0 \tag{15}$$

or

$$S^2 = \frac{KG - \omega_C{}^3}{3\omega_C} \tag{16}$$

Substituting (14) into (16) and solving,

$$S_{1,2} = \pm j\sqrt{3}\omega_C \tag{17}$$

Letting ω_n represent the frequency where the root locus crosses the imaginary axis, then

$$\omega_C = \frac{\omega_n}{\sqrt{3}} \tag{18}$$

See Figure 6. If the vertical axis is now shifted left by α units,

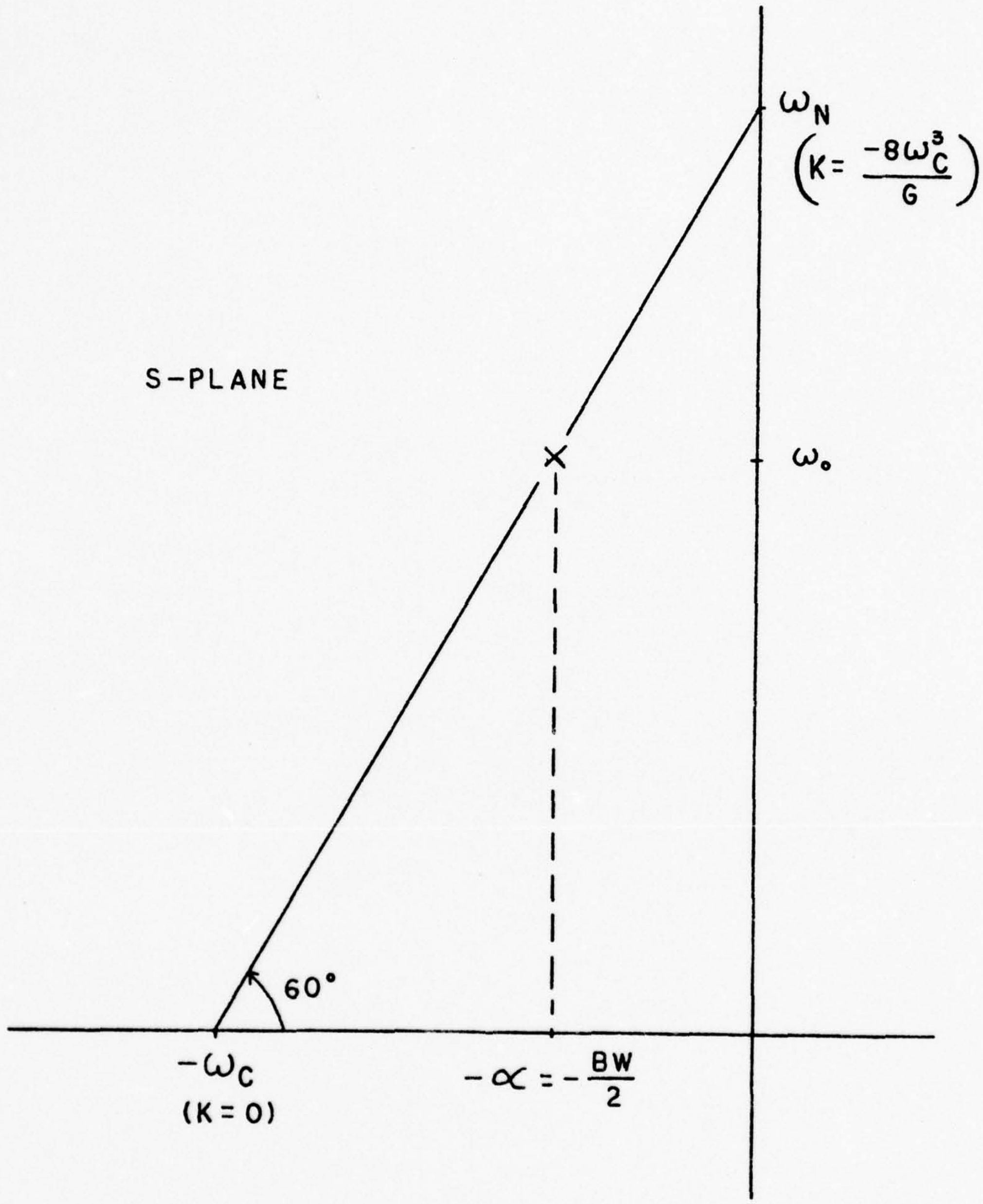


Figure 6. Root Locus Showing Crossing of Imaginary Axis

$$\omega_o = \omega_n(\text{shifted}) = \sqrt{3}(\omega_C - \alpha) \tag{19}$$

thus

$$\omega_C = \frac{\omega_o}{\sqrt{3}} + \frac{BW}{2} \tag{20}$$

and K becomes

$$K = \frac{-8(\omega_C - \frac{BW}{2})^3}{G} \tag{21}$$

or alternately

$$K = \frac{-8(\frac{\omega_o}{\sqrt{3}})^3}{G} \tag{22}$$

Now the open-loop poles and feedback gain, and hence the closed-loop pole locations, are given in terms of the desired filter parameters and the midband gain of the three-stage amplifier.

D. K IN TERMS OF CIRCUIT VALUES

Consider the diagram in Figure 7 in which the amplifier K has been replaced by a resistor of conductance G_f, and source resistance R_s and load conductance Y_L have been added for practicality.

The Y-parameters for the amplifier network are:

$$\begin{aligned} Y_{11A} &= Y_{IN} \qquad & Y_{12A} &= 0 \\ Y_{21A} &= Y_T \qquad & Y_{22A} &= Y_0 \end{aligned} \tag{23}$$

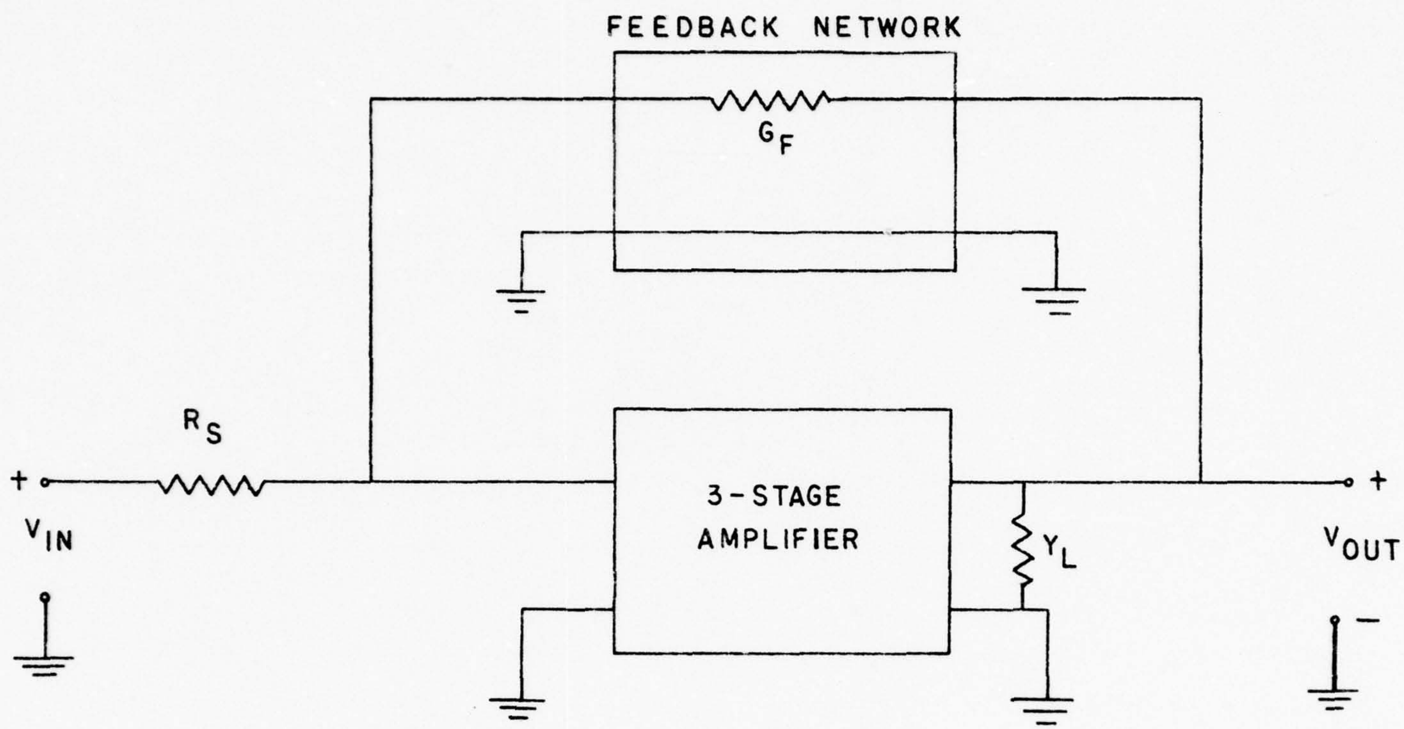


Figure 7. Three-Stage Amplifier With Resistive Feedback

where Y_{IN} is the input admittance and Y_0 is the output admittance, including Y_L, and the reverse transfer admittance is assumed negligible. Y_T is the forward transfer admittance of the three-stage amplifier. The Y-parameters of the feedback network are:

$$\begin{aligned} Y_{11F} &= G_f & \qquad Y_{12F} &= -G_f \\ Y_{21F} &= -G_f & \qquad Y_{22F} &= G_f \end{aligned} \tag{24}$$

Since the two networks are in parallel, the Y-parameters may be added to give total parameters of

$$\begin{aligned} Y_{11} &= Y_{IN} + G_f & \qquad Y_{12} &= -G_f \\ Y_{21} &= Y_T - G_f & \qquad Y_{22} &= Y_0 + G_f \end{aligned} \tag{25}$$

See Figure 8 for the total equivalent block diagram. The determinant of (25) is

$$\Delta Y = Y_{IN}Y_0+G_f(Y_{IN}+Y_0+Y_T) \tag{26}$$

The general form for the input impedance of such a network is (25, p. 28)

$$Z_{IN} = \frac{Y_L+Y_{22}}{Y_{11}Y_L+\Delta Y} \tag{27}$$

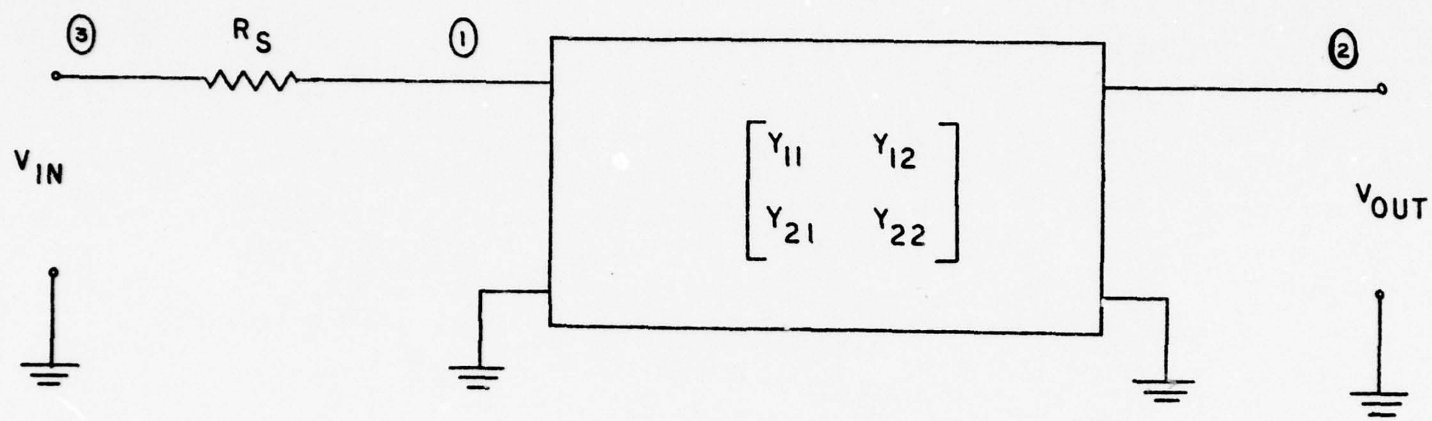


Figure 8. Total Equivalent Diagram For Three-Stage Amplifier With Resistive Feedback

But since Y_L is considered a part of Y_0, this reduces to

$$Z_{IN} = \frac{Y_{22}}{\Delta Y} \tag{28}$$

The voltage transfer function from ① to ② is (25, p. 28)

$$A_{v_{12}} = \frac{-Y_{21}}{Y_L+Y_{22}} = \frac{-Y_{21}}{Y_{22}} \tag{29}$$

and the voltage transfer function from ③ to ② is

$$A_{v_{32}} = \frac{A_{v_{12}} Z_{IN}}{R_s+Z_{IN}} \tag{30}$$

Substituting (28) into (30):

$$A_{v_{32}} = \frac{-Y_{21}}{R_s \Delta Y+Y_{22}} \tag{31}$$

which can be written as

$$A_{v_{32}} = \frac{-(Y_T-G_f)}{R_s(Y_{IN}Y_0+G_f(Y_{IN}+Y_0+Y_T))+Y_0+G_f} \tag{32}$$

$A_{v_{oL}}$, the open-loop voltage transfer function of the three-stage amplifier alone, is (once again Y_L is assumed a part of Y_0):

$$A_{v_{oL}} = \frac{-Y_T}{Y_0} \tag{33}$$

Combining (33) with (11) gives

$$Y_T = \frac{-G}{(S+\omega_C)^3} Y_0 \tag{34}$$

Substituting (34) into (32),

$$A_{v_{32}} = \frac{\dfrac{+GY_0}{(S+\omega_C)^3} + G_f}{R_s\left[Y_{IN}Y_0 + G_f\left(Y_{IN}+Y_0 - \dfrac{GY_0}{(S+\omega_C)^3}\right)\right] + Y_0 + G_f} \tag{35}$$

which can be written as

$$A_{v_{32}} = \frac{\left[GY_0 + G_f(S+\omega_C)^3\right] / \left[R_s Y_{IN} Y_0 + G_f R_s (Y_{IN}+Y_0) + Y_0 + G_f\right]}{(S+\omega_C)^3 - \dfrac{G\ Y_0\ G_f\ R_s}{\left[R_s Y_{IN} Y_0 + G_f R_s (Y_{IN}+Y_0) + Y_0 + G_f\right]}} \tag{36}$$

Since the poles of the transfer function are determined by the denominator, it is possible to compare the denominator of (36) with the denominator of (10), assuming Y_{IN} and Y_0 are real and constant, to get

$$K = \frac{Y_0\ G_f\ R_s}{R_s Y_{IN} Y_0 + G_f R_s (Y_{IN}+Y_0) + Y_0 + G_f} \tag{37}$$

Solving for G_f,

$$G_f = \frac{K\ Y_0\left[R_s Y_{IN} + 1\right]}{R_s\left[Y_0 - K(Y_{IN}+Y_0) - \dfrac{K}{R_s}\right]} \tag{38}$$

Substituting (14) into (38),

$$G_f = \frac{-8\omega_C{}^3 Y_0 (R_s Y_{IN}+1)}{G\ R_s\left[Y_0 + \frac{8\omega_C{}^3}{G}(Y_{IN}+Y_0) + \frac{8\omega_C{}^3}{G\ R_s}\right]} \tag{39}$$

Note that G is negative so that G_f is positive. (It has been tacitly assumed that all zeroes are far enough away to be neglected. This will be proved later.)

The feedback resistance is now in terms of the midband gain, input and output admittances of the amplifier, and the source resistance -- all terms that may be readily calculated. All that remains is to place the open-loop poles in the desired position, namely at $S_{1,2,3} = -\omega_C$.

E. OPEN-LOOP POLE, ω_C, IN TERMS OF CIRCUIT PARAMETERS

Consider the three-stage amplifier in Figure 9. Emitter resistors have been introduced to help stabilize the DC bias conditions since the three stages are DC-coupled; resistors R_1, R_2, and R_3 will be utilized in setting the desired open-loop pole locations. The AC equivalent circuit, using the hybrid-π model for the transistors (18-pp. 244-245), is shown in Figure 10. The extrinsic terminal-to-terminal capacitances are ignored. These capacitances are caused by interlead capacitance when a transistor is mounted on a header and leads are attached. Additionally, the extrinsic capacitance from

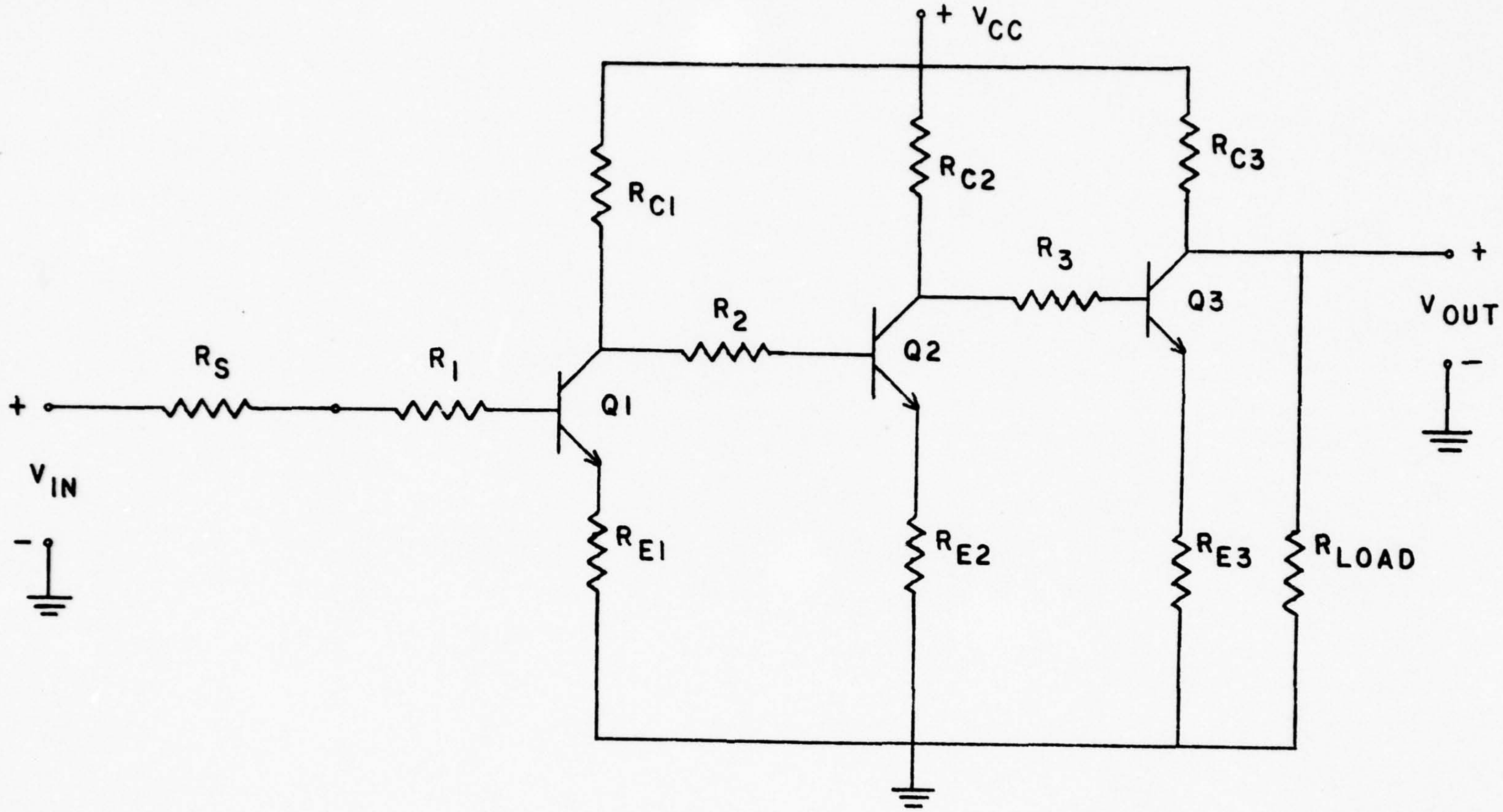


Figure 9. Three-Stage Amplifier

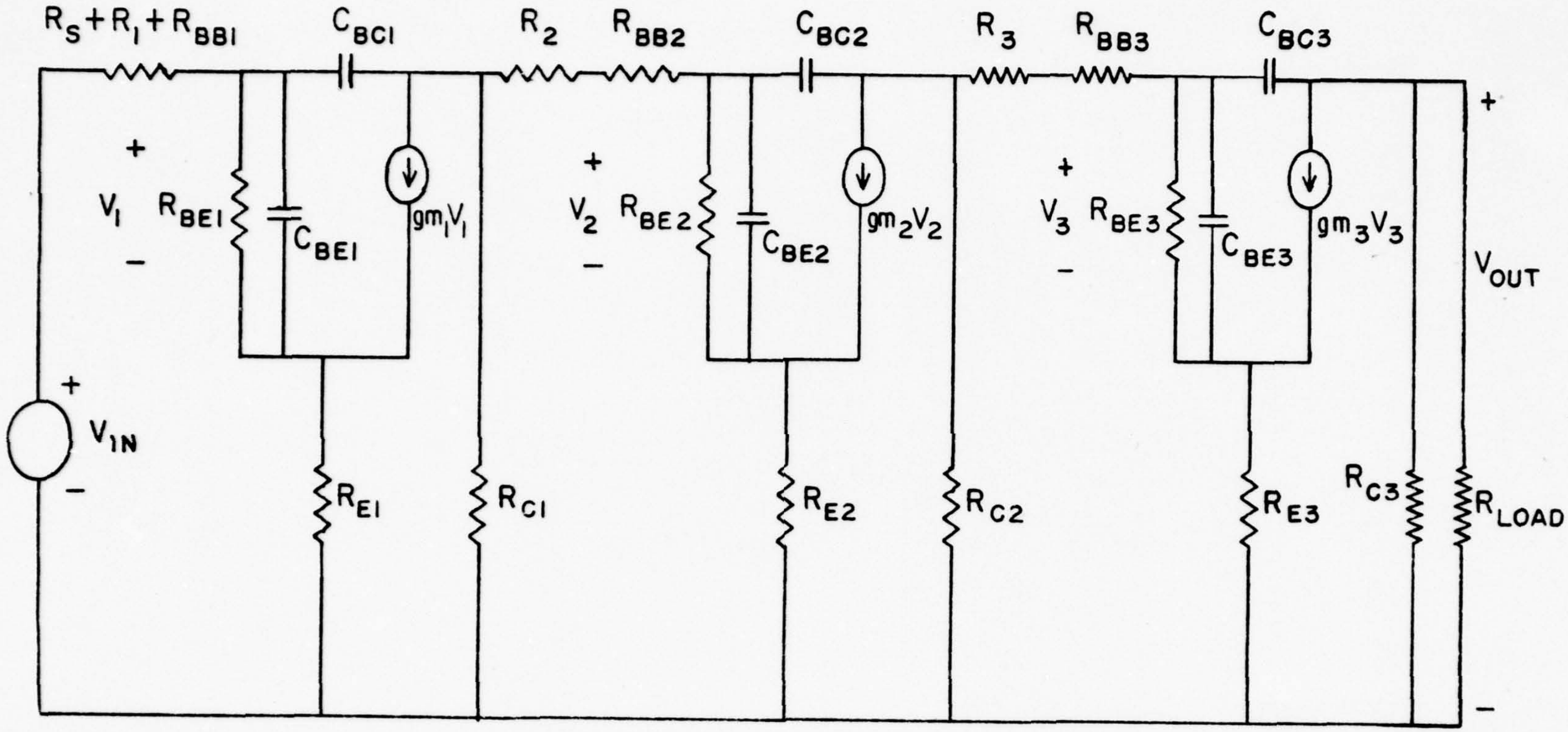


Figure 10. AC Equivalent Circuit Of Three-Stage Amplifier

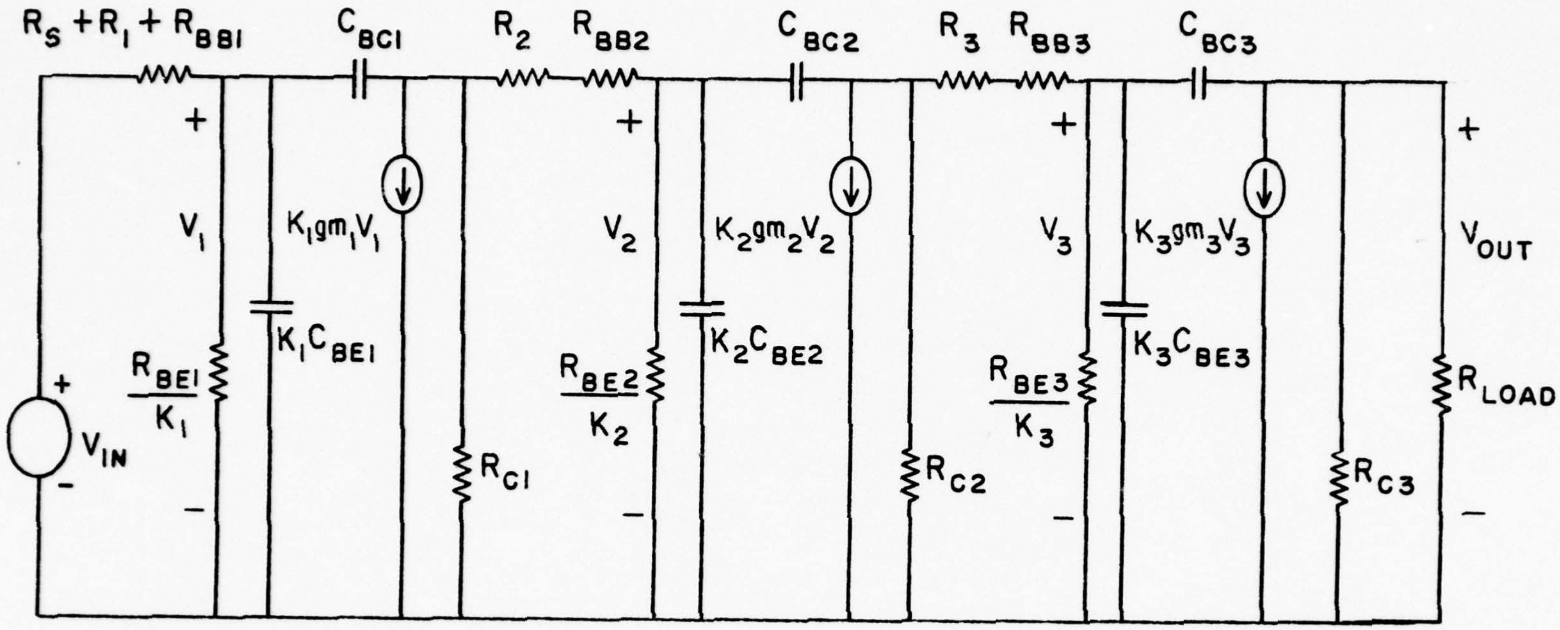
base to collector includes the incremental space-charge capacitance associated with the portions of the collector junction that lie outside the active base region. This portion of the collector junction is sometimes called the "overlap diode", and the corresponding component of the extrinsic capacitance is the "overlap capacitance" (17, pp. 383-384).

By using the method of absorbing an external emitter resistor as presented by Thornton (20, pp. 52-56), Figure 10 may be approximated by Figure 11, where gm, R_{BE}, and C_{BE} for each stage have been modified by $K = \frac{1}{1+gmR_E}$. Now applying the Miller effect (19, pp. 359-361), Figure 11 may be further simplified to Figure 12. The small Miller output capacitances across the terminals of the current sources have been neglected in Figure 12 since their value is approximately equal to C_{BC} which is small compared to other circuit elements.

The open-loop poles may now be set by noting that for each stage (18, pp. 256-257)

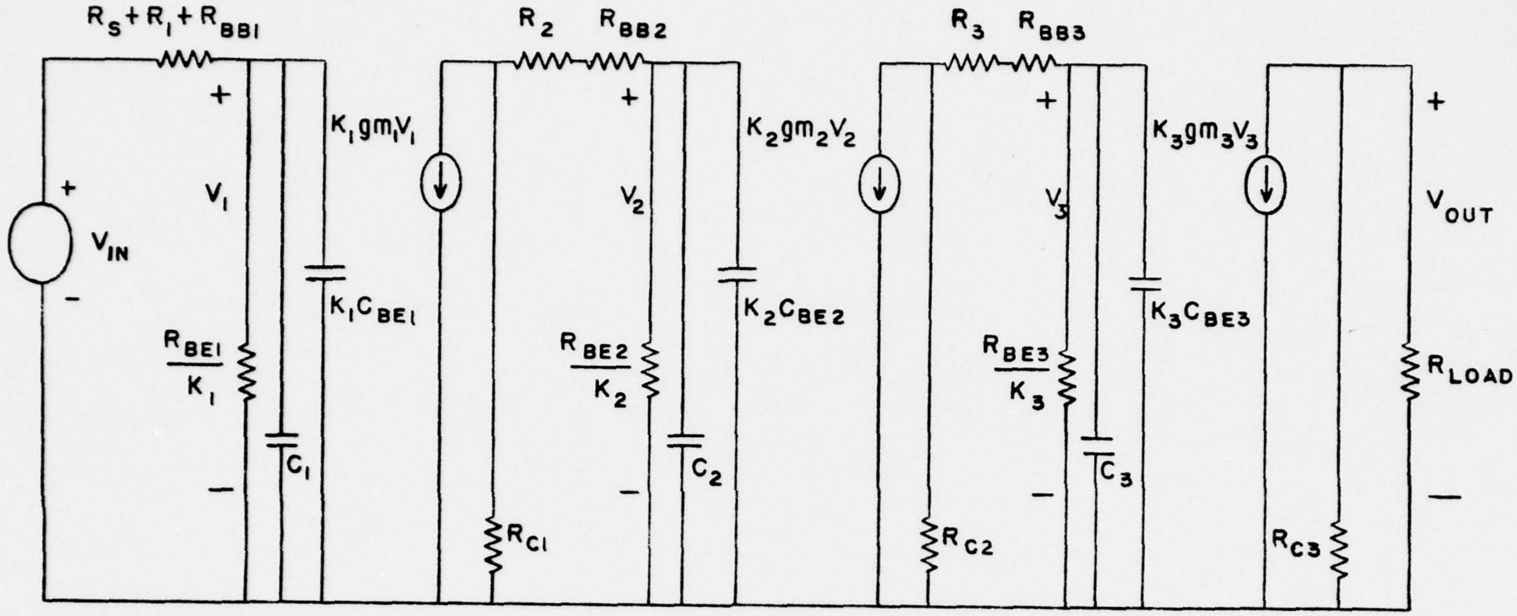
$$\omega_n = \frac{1}{R_{Tn}C_{Tn}} \qquad n = a, b, c \tag{40}$$

where C_T is the total capacitance on the input of each stage, and R_T is the total resistance in parallel with that capacitance, and ω_n is the upper 3-dB frequency when no excess phase shift is considered (see Appendix C). Letting K_θ represent the phase-correction consent, then



$$K_n = \frac{1}{1 + gm_n R_{E_n}} \qquad n = 1,2,3$$

Figure 11. Three-Stage Amplifier After Absorbing Emitter Resistors



$$C_1 = C_{BC1}\left(1 + K_1 gm_1\left[R_{C1} \,||\, \left\langle R_2 + R_{BB2} + \frac{R_{BE2}}{K_2}\right\rangle\right]\right)$$

$$C_2 = C_{BC2}\left(1 + K_2 gm_2\left[R_{C2} \,||\, \left\langle R_3 + R_{BB3} + \frac{R_{BE3}}{K_3}\right\rangle\right]\right)$$

$$C_3 = C_{BC3}\left(1 + K_3 gm_3\left[R_{C3} \,||\, R_{LOAD}\right]\right)$$

Figure 12. Simplified Three-Stage Amplifier After Applying Miller Effect

$$\omega_a = \frac{\omega_A}{K_\theta} = \frac{1}{\left[\frac{R_{BE1}}{K_1} \,\Big|\Big|\, (R_s+R_1+R_{BB1})\right]\left[C_1+K_1C_{BE1}\right]} \tag{41}$$

$$\omega_b = \frac{\omega_B}{K_\theta} = \frac{1}{\left[\frac{R_{BE2}}{K_2} \,\Big|\Big|\, (R_{C1}+R_2+R_{BB2})\right]\left[C_2+K_2C_{BE2}\right]} \tag{42}$$

$$\omega_c = \frac{\omega_C}{K_\theta} = \frac{1}{\left[\frac{R_{BE3}}{K_3} \,\Big|\Big|\, (R_{C2}+R_3+R_{BB3})\right]\left[C_3+K_3C_{BE3}\right]} \tag{43}$$

Substituting the values for C_1, C_2, and C_3 and solving these expressions for R_1, R_2, and R_3:

$$R_3 = \frac{1}{\frac{\omega_C}{K_\theta}\left(K_3C_{BE3}+C_{BC3}\left[1+K_3gm_3\,\frac{R_{C3}R_{LOAD}}{R_{C3}+R_{LOAD}}\right]\right) - \frac{K_3}{R_{BE3}}} - R_{C2} - R_{BB3} \tag{44}$$

$$R_2 = \frac{1}{\frac{\omega_B}{K_\theta}\left(K_2C_{BE2}+C_{BC2}\left[1+K_2gm_2\frac{R_{C2}\langle R_3+R_{BB2}+\frac{R_{BE3}}{K_3}\rangle}{R_{C2}+R_3+R_{BB2}+\frac{R_{BE3}}{K_3}}\right]\right) - \frac{K_2}{R_{BE2}}} - R_{C1} - R_{BB2} \tag{45}$$

$$R_1 = \frac{1}{\frac{\omega_A}{K_\theta}\left(K_1 C_{BE1} + C_{BC1}\left[1 + K_1 gm_1 \frac{R_{C1} < R_2 + R_{BB2} + \frac{R_{BE2}}{K_2} >}{R_{C1} + R_2 + R_{BB2} + \frac{R_{BE2}}{K_2}}\right]\right) - \frac{K_1}{R_{BE1}}} \tag{46}$$

$$- R_s - R_{BB1}$$

Assuming that R_C, R_E, and the transistors (and hence the hybrid-π parameters) are all chosen arbitrarily, R_1, R_2, and R_3 may now be used to set the open-loop poles to $\omega_A = \omega_B = \omega_C$ as required by the desired center frequency and bandwidth.

Returning to (38), it is noted that for $G_f > 0$, then

$$Y_{IN} < Y_0 \left(\frac{1-K}{K}\right) - \frac{1}{R_s} \tag{47}$$

To insure this condition, as well as the condition that Y_{IN} is real and constant from (37), an emitter follower is added at the input of the three-stage amplifier. Since an emitter follower employs a large amount of feedback through the emitter resistor, its gain is practically frequency independent and thus will not affect the placement of the open-loop poles as previously derived. The output admittance of an emitter follower is (19, p. 252)

$$Y_{OUT} = h_{oe} + \frac{1+h_{fe}}{h_{ie}+R_s} \tag{48}$$

where R_s is the source resistance. In (46), R_s should now be replaced by $1/Y_{OUT}$. From (47), for $Y_{IN}>0$, then

$$R_s > \frac{1}{Y_0}\left(\frac{K}{1-K}\right) \tag{49}$$

where R_s is the output impedance of the driving source. To achieve this condition, it may be necessary to add some resistance in series with the base of the emitter follower.

F. ZERO LOCATIONS

Recall that it was assumed that zero locations were far enough away to be neglected in the dominant-pole derivation. From (36), the transfer function will have three zeroes. These are best calculated by considering Figure 11. If for some value of S, say S_3, V_{OUT} is zero, then the current through C_{BC3} is $K_3 gm_3 V_3$. Hence

$$S_3 C_{BC3} V_3 = K_3 gm_3 V_3 \tag{50}$$

and the zero is

$$S_3 = \frac{K_3 gm_3}{C_{BC3}} \tag{51}$$

Similarily, the other zeroes are

$$S_2 = \frac{K_2 gm_2}{C_{BC2}} \tag{52}$$

and

$$S_1 = \frac{K_1 gm_1}{C_{BC1}} \tag{53}$$

Since C_{BC} is small (typically 2-4 pf), all the zeroes are positive and well into the right-half-plane. To get an idea of the magnitude of the zeroes, consider gm_1=40 MMHOS and K_1=.048 (i.e., R_{E1}=500 OHMS), with C_{CB1} = 4 pf. Then $S_1 = 4.8 \times 10^8$ radians/sec. Since gm_1 and K_1 are often larger, the zeroes are then even further into the right-half-plane. Thus the assumption that the zeroes may be neglected is justified.

G. OTHER POLES

By examining Figure 10, six poles are expected in the voltage transfer function of the three-stage amplifier since there are six capacitors and no capacitor loops. However, only three poles were used in the derivation. The justification for this can be seen in Figure 12 which effectively contains only three capacitors. The missing poles are due to the Miller output capacitances across the terminals of the three current sources which were neglected. The value of this capacitance is (19, pp. 359-361)

$$C_{MILLER} = C_{BC}\left(\frac{G_V - 1}{G_V}\right) \tag{54}$$

Where G_V is approximately the voltage gain of the stage and

$$G_V >> 1 \tag{55}$$

Hence

$$C_{MILLER} \approx C_{BC} \tag{56}$$

C_{CB} is small (typically 2-4 pf) and the total resistance across its terminals is approximately (considering the middle stage)

$$R_{C2}||(R_3+R_{BB3}+\frac{R_{BE3}}{K_3}) \approx R_{C2} \tag{57}$$

The 3-dB frequency associated with this capacitance is (approximately)

$$\omega_{MILLER} \approx \frac{1}{C_{BC2}R_{C2}} \tag{58}$$

But in Figure 12,

$$C_{TC} = C_3+K_3C_{BE3} >> C_{BC2} \tag{59}$$

and

$$R_{TC} = \frac{R_{BE3}}{K_3}||(R_{BB3}+R_3+R_{C2}) >> R_{C2} \tag{60}$$

Thus

$$\omega_C = \frac{1}{R_{TC}C_{TC}} << \omega_{MILLER} \tag{61}$$

And hence ω_{MILLER} may be neglected. This is verified for the test circuit of Figure 13 by a computer program (17-Appendix C) that finds the natural frequencies of a network given the node voltage equations. The results are listed in Table II.

TABLE II

POLES OF TEST CIRCUIT
(Figure 13)

	Real	Imaginary
s_1	-0.139082×10^{6}	0.4463298×10^{7}
s_2	-0.139082×10^{6}	-0.4463298×10^{7}
s_3	-0.8717343×10^{7}	0.0
s_4	-0.1799509×10^{10}	0.2647827×10^{7}
s_5	-0.1799509×10^{10}	-0.2647827×10^{7}
s_6	-0.1789797×10^{10}	0.0
s_7	-0.1674205×10^{10}	0.1463698×10^{8}
s_8	-0.1674205×10^{10}	-0.1463698×10^{8}
s_9	-0.635348×10^{8}	0.0
s_{10}	-0.3523957×10^{8}	0.0

NOTE: All values in radians/second.

III. RESULTS

A. TEST CIRCUIT

The design procedures were tested with the circuit of Figure 13. Design center frequency was 700 kHz with a bandwidth of 50 kHz. R_E and R_C were arbitrarily chosen to be 200 ohms and 2k ohms, respectively. Identical 2N3904 transistors were selected to have an h_{fe} of 150 by using a curve tracer. Transistor data sheets (24) were used to determine the hybrid-π parameters at V_{CE} = 10 VDC and I_C = 1ma at room temperature (see Appendix A), and are listed in Table III. As a first approximation, the case of no excess phase shift is considered, that is, arbitrarily let $K_\theta = 1$. Computer program "Filter" (Appendix B) was used to determine R_1, R_2, R_3, and $R_F = \frac{1}{G_f}$. Transistor Q1 is an emitter follower used to allow the DC bias current from the collector of Q5, through R_F, R_{s1}, and R_D to ground and not load down the signal generator. The AC output impedance of the emitter follower formed by Q1 is approximately 20 ohms and is added to R_{s1} to form R_s used in the design procedure. Zener diodes ZD1 and ZD2 are used for coupling stages two and three with R_{C1B} and R_{C2B} providing the necessary zener currents. Due to the inability to select precise values for ZD1 and ZD2, it was necessary to apply a source at point A and adjust the Q-points of the transistors individually by varying R_{C1B}, R_{C2B}, and R_B

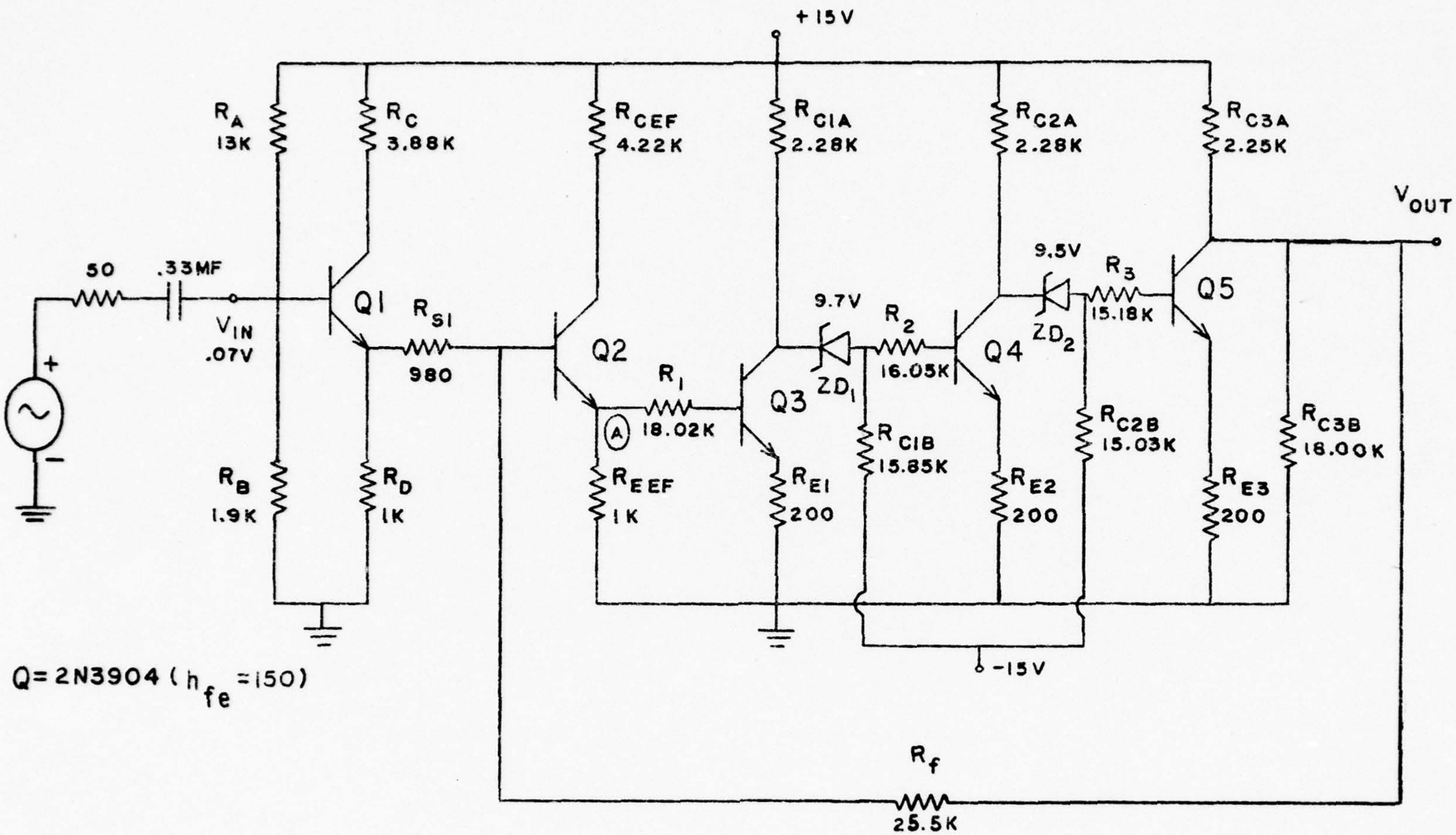


Figure 13. Test Circuit With Design F_o=700kHz And BW=50kHz

TABLE III

HYBRID-π PARAMETER OF 2N3904 TRANSISTOR

(V_{CE}=10 VDC, I_C=1 ma, h_{fe}=150)

R_{BE}	3.75	K ohms
R_{BB}	100	ohms
C_{BC}	3	pF
C_{BE}	25	pF
gm	40	MMHOS

(to control Q2). Thus the parallel combination of R_{C1A} and R_{C1B} and of R_{C2A} and R_{C2B} does not equal 2K as desired (1.99K and 1.97K respectively). This is a possible source of slight errors in the results. The frequency-response appears in Figure 14. Note that the center frequency was 485 kHz with a bandwidth of 88 kHz. Results of an ECAP simulation program of the circuit is shown in Figure 15. Since these results (center frequency = 710 kHz, bandwidth = 49 kHz) agree well with the design, errors in calculating the hybrid-π parameters from the transistor data sheet, effects of extrinsic capacitances, or wrong K_θ are suspected.

B. EFFECTS OF EXTRINSIC CAPACITANCE

In Figure 16, extrinsic capacitances have been added across the terminals of the hybrid-π model of the transistor. For low-power transistors, C_{be} and C_{ce} are approximately ½ pF, due mostly to the header and lead capacitance. However, C_{bc} may be of the order of 2 pF (for low-power transistors) to 200 pF (for high-power units) since it consists mostly of overlap-diode capacitance (21, pp. 102-103). Since C_{bc} is approximately across C_{BC} (separated only by R_{BB} which is very small), C_{bc} adds directly to C_{BC}. Although C_{bc} is small, its importance is magnified by the Miller effect.

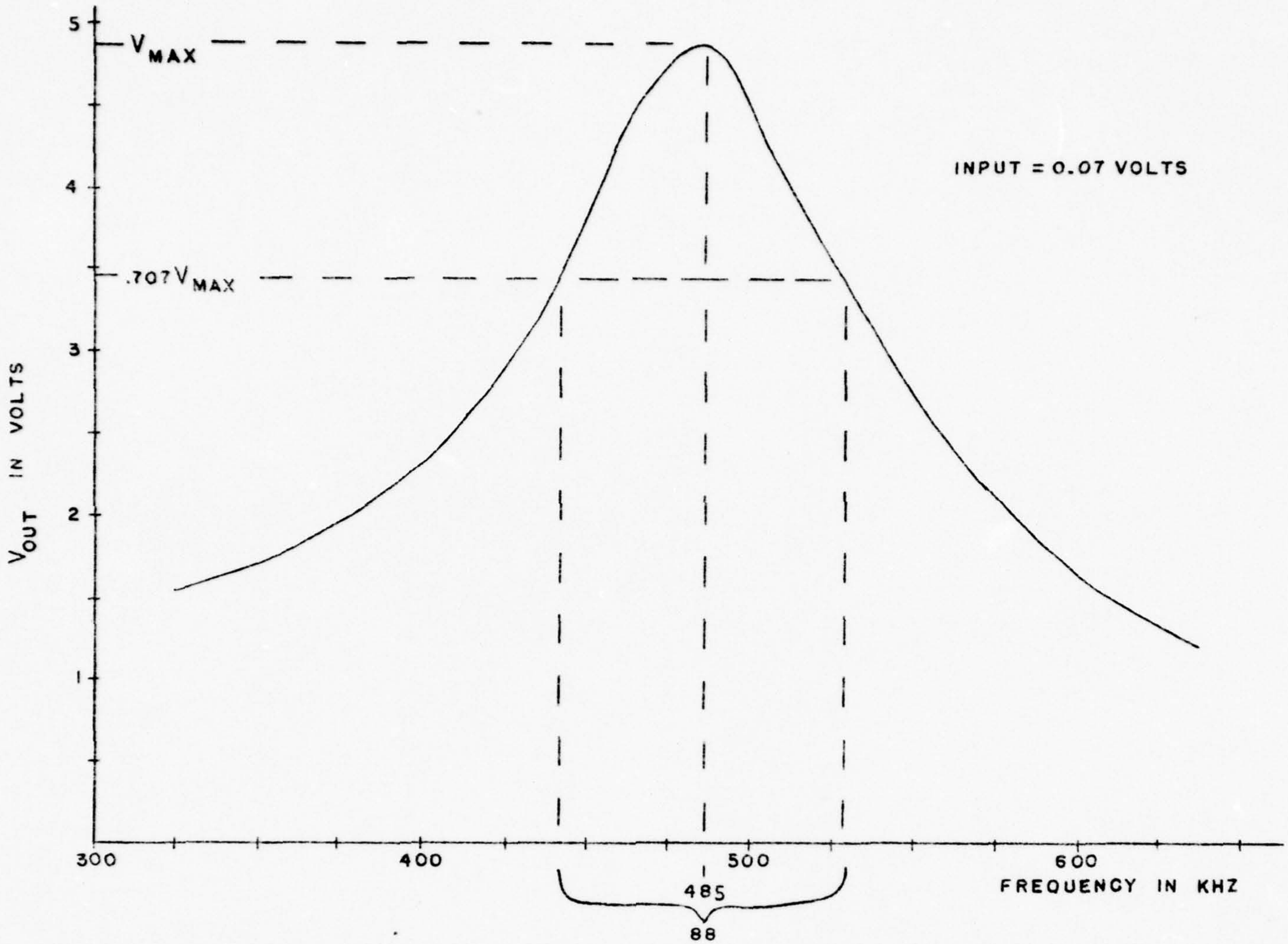


Figure 14. Frequency Response Of Test Circuit In Figure 13

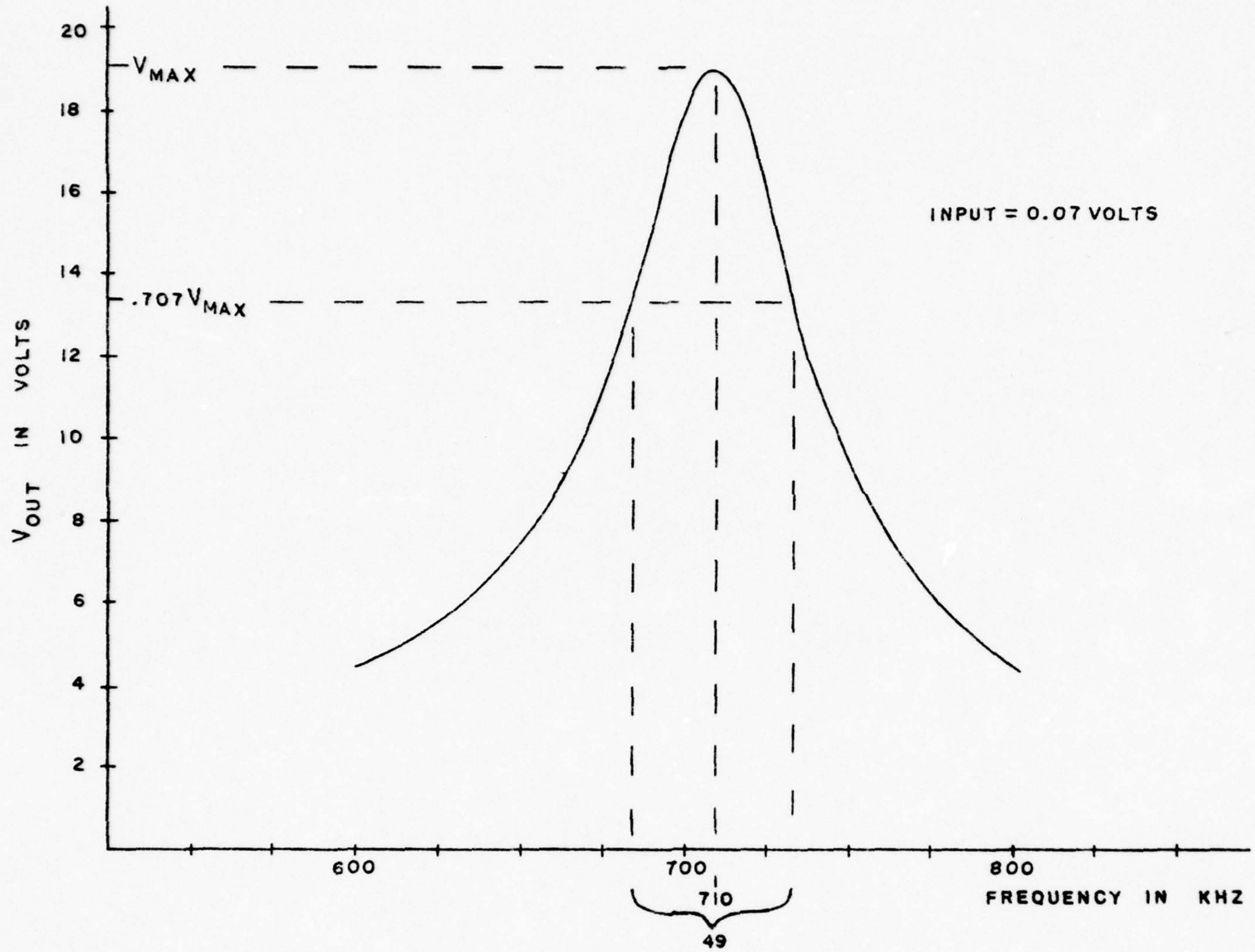


Figure 15. Frequency Response of ECAP Simulation Of Test Circuit In Figure 13

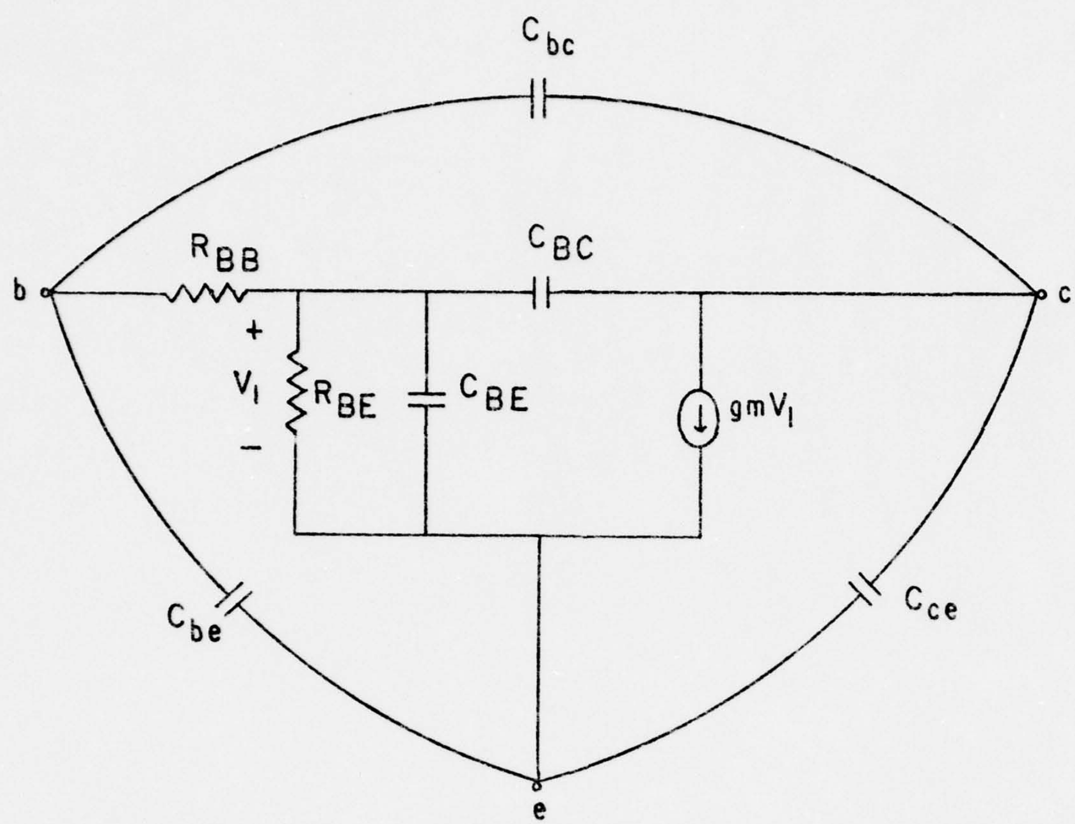


Figure 16. Hybrid-π Model Of Transistor With Extrinsic Capacitances

Using the results of the test circuit of Figure 13, and assuming only C_{BC} is incorrect, it is easy to work backwards and calculate the value of C_{BC} to give the results actually achieved. Since

$$\omega_o = 485x10^3x2\pi = 3.0473449x10^6 \ \frac{\text{radians}}{\text{sec}} \tag{62}$$

and

$$BW = 88x10^3x2\pi = 5.5292031x10^5 \ \frac{\text{radians}}{\text{sec}} \tag{63}$$

Then from (20)

$$\omega_C = \frac{\omega_o}{\sqrt{3}} + \frac{BW}{2} = 2.0358455x10^6 \ \frac{\text{radians}}{\text{sec}} \tag{64}$$

Substituting (64) into (44) and solving for C_{BC3},

$$C_{BC3} = 4.08 \text{ pF} \tag{65}$$

Which differs from the design value of 3 pf by 1.08 pF, comparing favorably with the approximate value of C_{bc} to be expected.

C. RESULTS CONSIDERING EXCESS PHASE SHIFT

The test circuit of Figure 13 was based on $K_\theta = 1$, that is, no excess phase shift. If it is assumed that $K_\theta = 0.7$ for this transistor, the values of R_1, R_2, R_3 and R_F in Figure 13 are modified to 10.886K, 8.925K, 8.388K and 41.188K ohms respectively. The frequency response for this circuit appears in Figure 17. Note the improvement in center frequency to 667kHz, which is within 5% of the desired. However, the bandwidth increased to 135kHz, as compared to the 50kHz desired. Results using the actual measured value of K_θ for each transistor were not investigated.

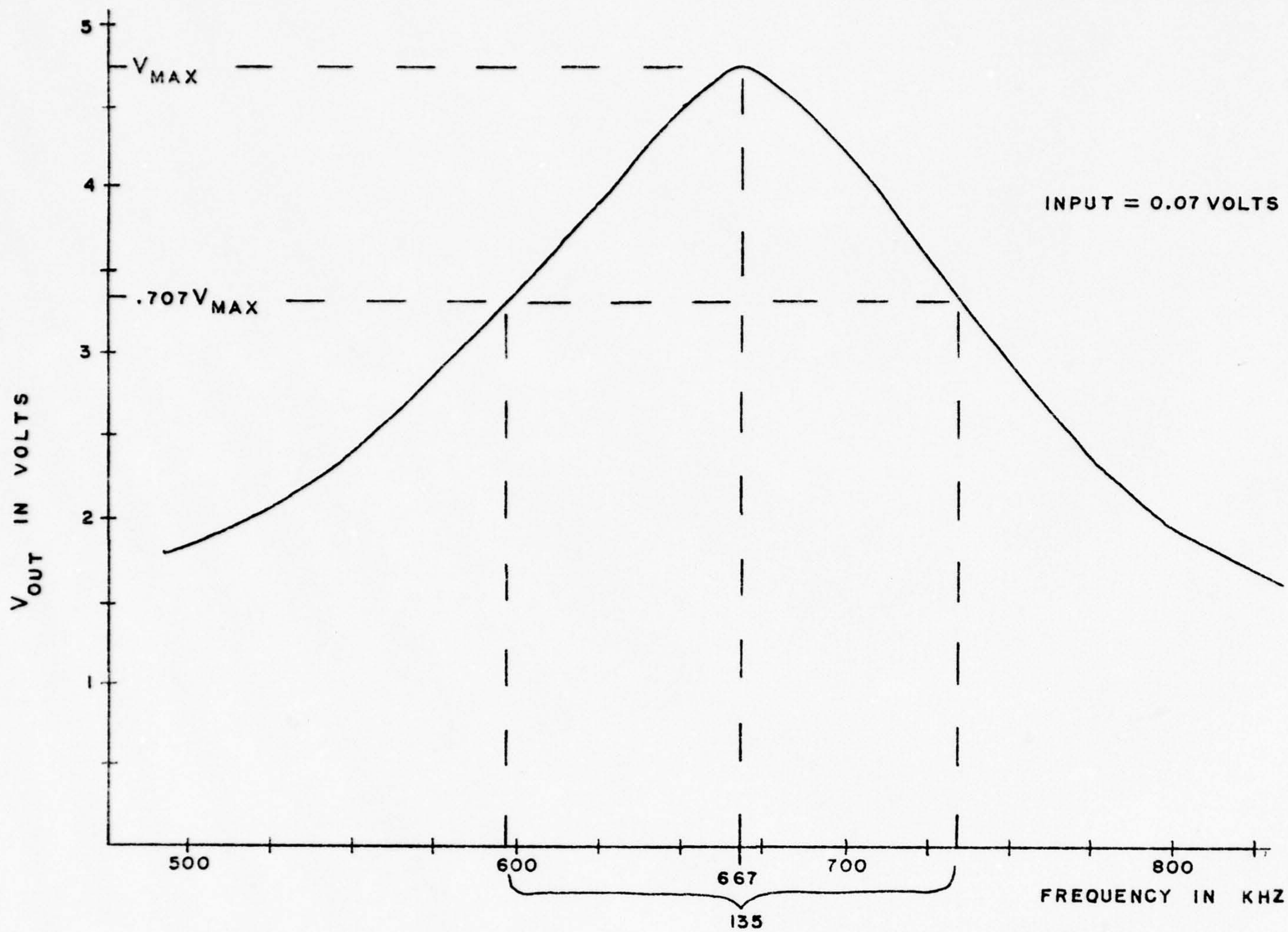


Figure 17. Frequency Response Of Test Circuit Considering Excess Phase Shift

IV. LIMITATIONS

The design of the bandpass filter proposed here depends upon the precise knowledge of the transistor hybrid-π parameters; the usefulness of a practical application of such a design is severely limited since transistor parameters vary widely, even among transistors of the same manufacturer's lot. This would necessitate individual measurement of at least some of the hybrid-π parameters of the transistors to be used.

The DC bias circuit used in Figure 13 is not necessarily the best method and was only used to test the AC characteristics investigated in this work. More study in this area is necessary to alleviate the stage-by-stage adjustments that were necessary to achieve proper bias conditions.

It was noted that a small change in C_{BC} had a profound effect on the center frequency. Since C_{BC} is proportional to $V_{CB}^{-1/3}$ for a graded-junction device (17, p. 428), it is expected that the design is very sensitive to bias instability, although no study was done in that area.

The center frequency and bandwidth achievable obviously depends on the ability to place the open-loop poles, ω_C, and retain sufficient amplifier gain so that the feedback gain, K, remains less than unity so that passive feedback may be used. If a desired center

frequency and bandwidth cannot be achieved (as evidenced by a negative R_1, R_2, R_3 or R_F), it may be necessary to change the arbitrarily chosen R_E, R_C, or transistor type.

To get a rough idea of the maximum and minimum ω_o obtainable with a given set of parameters, it is useful to examine the last stage of the amplifier. For convenience assume $R_{LOAD} \to \infty$ and BW = 0 so that from (20)

$$\omega_C = \frac{\omega_o}{\sqrt{3}} \tag{66}$$

Thus (44) becomes

$$R_3 = \frac{1}{\frac{\omega_o}{\sqrt{3}}\left[K_3 C_{BE3} + C_{BC3}(1+K_3 gm_3 R_{C3})\right] - \frac{K_3}{R_{BE3}}} - R_{C2} - R_{BB3} \tag{67}$$

Maximum ω_o is obtained when $R_3 = 0$ so that

$$\frac{\omega_{o\ max}}{\sqrt{3}} = \frac{1}{(R_{C2}+R_{BB3})\left[\left\langle K_3 C_{BE3} + C_{BC3}(1+K_3 gm_3 R_{C3})\right\rangle - \frac{K_3}{R_{BE3}}\right]} \tag{68}$$

Thus decreasing R_{C2}, R_{C3}, or choosing a transistor with smaller C_{BE} and C_{BC} would tend to increase $\omega_{o\ max}$. The effect of decreasing K_3, obtained by increasing R_E since

$$K_3 = \frac{1}{1+gm_3 R_{E3}} \tag{69}$$

is not so evident and must be examined using the specified parameters involved. Generally, however, increasing R_E will increase $\omega_{o\ max}$.

The minimum value of ω_o is more difficult to analyze. Figure 18 is the general form of R_3 vs. ω_o from (67). It would appear that $\omega_{o\ min}$ is obtained when $R_3 \rightarrow \infty$. However, as $R_3 \rightarrow \infty$, the gain of the amplifier approaches zero. For the feedback network to remain passive, the magnitude of K must be less than unity. From (21)

$$|K| = \left|\frac{-8\omega_C^{\,3}}{G}\right| < 1 \tag{70}$$

But since

$$G = ABC\omega_C^{\,3} \tag{71}$$

(70) becomes

$$8 < ABC \tag{72}$$

Assuming equal midband gain for the three stages (a very gross assumption but useful for this analysis!), then

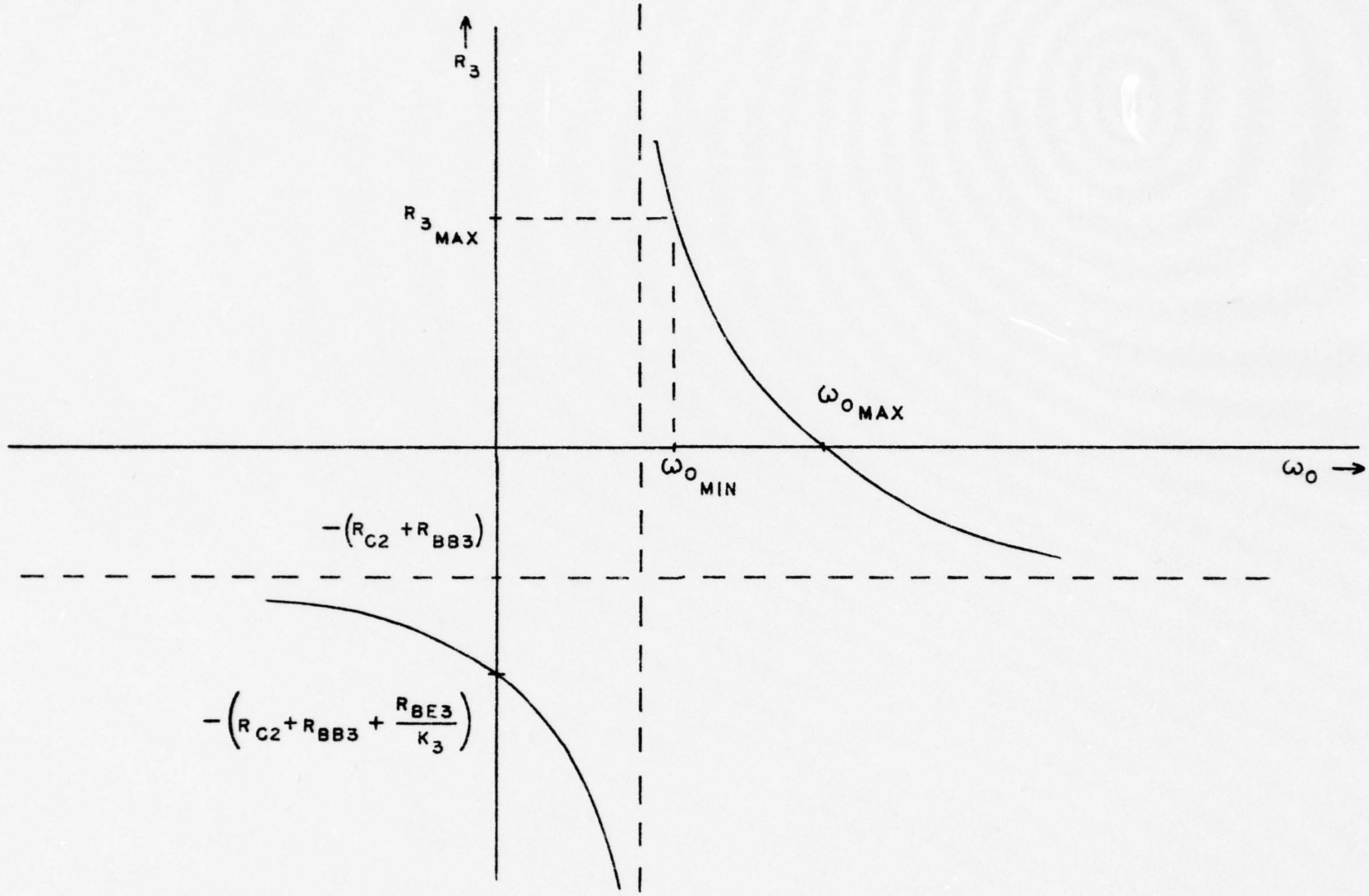


Figure 18. General Form Of R_3 Vs. ω_o

$$2 < \text{gain one stage} \approx \frac{\frac{R_{BE3}}{K_3} gm_3 K_3 RC_3}{\frac{R_{BE3}}{K_3} + R_3} \tag{73}$$

From which

$$R_{3\ max} < \tfrac{1}{2} R_{BE3} gm_3 R_{C3} - \frac{R_{BE3}}{K_3} \tag{74}$$

This maximum value of R_3 is now used to find the minimum ω_o as in Figure 18. To increase $R_{3\ max}$ (and to insure that it is not negative), it is desired that

$$\tfrac{1}{2} R_{BE3} gm_3 R_{C3} > \frac{R_{BE3}}{K_3} \tag{75}$$

But

$$\frac{1}{K_3} = \frac{1+gm_3 R_{E3}}{1} \approx gm_3 R_3 \tag{76}$$

since $gm_3 R_{E3} >> 1$. Thus (75) becomes

$$\tfrac{1}{2} R_{BE3} gm_3 R_{C3} > R_{BE3} gm_3 R_{E3} \tag{77}$$

or

$$R_{C3} > 2R_{E3} \tag{78}$$

The greater this inequality, the smaller $\omega_{o\ min}$ obtainable with a given set of parameters. It should be noted

again that the relationships for $\omega_{o\ max}$ and $\omega_{o\ min}$ developed above examined only one stage of the three-stage amplifier and did not include interaction among the stages. Also gross assumptions were made. As such, these relationships should only be used to gain insight into what parameters should be changed if a desired center frequency cannot be obtained.

V. CONCLUSION

By considering the internal capacitances of a transistor, a bandpass filter may be developed utilizing no external capacitors, thereby reducing the amount of space necessary for an integrated circuit. The results of the test circuit were somewhat disappointing and highlight the limitations of the design. More study is necessary to lessen the sensitivity to extrinsic elements and to explore the effects of bias instability. The design developed here may have more appeal once integrated circuitry technology has developed a method to more accurately manufacture a device with given parameters (such as h_{fe} and C_{BC}) and insure only slight variations of those parameters from device to device throughout the production run.

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VITA

Lester Charles Roth was born on August 14, 1945 in Festus, Missouri. He received his primary education at Jefferson R-7 Elementary School near Festus, Missouri and his secondary education at Festus High School. In 1969 he received a B.S. in Electrical Engineering from the University of Missouri at Rolla and immediately entered the U.S. Army, Corps of Engineers.

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APPENDIX A

DETERMINATION OF HYBRID-π PARAMETERS OF 2N3904 TRANSISTOR

From transistor data sheets (24), Table IV is obtained. Following the method outlined by Gray (17, pp. 427-430), for the operating point of $I_C = 1$ ma, $V_{CE} = 10$ VDC, the value of gm at room temperature is found as:

$$gm = \frac{q}{kT} |I_C| \cong 40x1 = 40 \text{ MMHOS} \qquad \text{(A-1)}$$

By use of a transistor curve tracer, identical transistors are selected to have $\beta_o = 150$ at the desired q-point. Thus

$$R_{BE} = \frac{\beta_o}{gm} = \frac{150}{40} = 3.75 \text{ K ohms} \qquad \text{(A-2)}$$

The capacitance C_{BC} is approximately equal to C_{ob} given in Table IV but is dependent on the reverse bias voltage V_{CB}. For an npn graded-junction device, the proportionality is

$$C_{BC} \sim V_{CB}^{-1/3} \qquad \text{(A-3)}$$

Since

$$V_{CB} \approx V_{CE} = 10 \text{ volts} \qquad \text{(A-4)}$$

then

TABLE IV

VALUES FROM TRANSISTOR DATA SHEET

Small Signal Characteristic	Symbol	Value
Current-Gain-Bandwidth Product (I_C=10MA, V_{CE}=20VDC, f=100MHZ)	f_T	250 MHz (min)
Output Capacitance (V_{CB}=5.0VDC, I_C=0, f=100kHz)	C_{ob}	4.0 pF (max)
Small-Signal Current Gain (I_C=1MA, V_{CE}=10VDC, f=1.0kHz)	h_{fe}	100-400

$$C_{BC} = 4\left(\frac{5}{10}\right)^{+1/3} \approx 3 \text{ pF} \quad \text{(A-5)}$$

The expression for C_{BE} is

$$C_{BE} = \frac{gm}{2\pi f_T} - C_{BC} \quad \text{(A-6)}$$

However, since f_T is specified at I_C=10MA and V_{CE}=20VDC, gm at this current must first be determined as

$$gm \approx 40|I_C| = 40x10 = 400 \text{ MMHOS} \quad \text{(A-7)}$$

and

$$C_{BC} = 4\left(\frac{5}{20}\right)^{1/3} \approx 2.5 \text{ pF} \quad \text{(A-8)}$$

Thus at I_C=10MA and V_{CE}=20VDC,

$$C_{BE} = \frac{400x10^{-3}}{2\pi x250x10^{6}} - 2.5x10^{-12} \approx 252 \text{ pF} \quad \text{(A-9)}$$

Now since C_{BE} scales linearly with I_C,

$$C_{BE} = 252\left(\frac{1}{10}\right) \approx 25 \text{ pF} \quad \text{(A-10)}$$

at I_C=1MA and V_{CE}=20VDC. Lacking information of the relationship between C_{BE} and V_{CE}, assume C_{BE} does not

change significantly from V_{CE}=20VDC to V_{CE}=10VDC. Lacking any information on R_{BB}, assume

$$R_{BB} = 100\Omega \tag{A-11}$$

Note that this assumption is not critical since R_{BB} is small in comparison to typical values of R_1, R_2, and R_3 (usually greater than several K ohms). The hybrid-π values of the 2N3904 transistor used in the design of the test circuit are consolidated in Table III, page 35.

APPENDIX B

COMPUTER PROGRAM FILTER

Computer program "Filter" is used to calculate the value of the base resistances (R_1, R_2, and R_3) and R_F to achieve a desired filter characteristics. Although the program assumes equal values of collector resistors, emitter resistors, and transistor parameters, it may be easily modified to allow variation of these elements. Figure 19 is the circuit diagram for which the program applies.

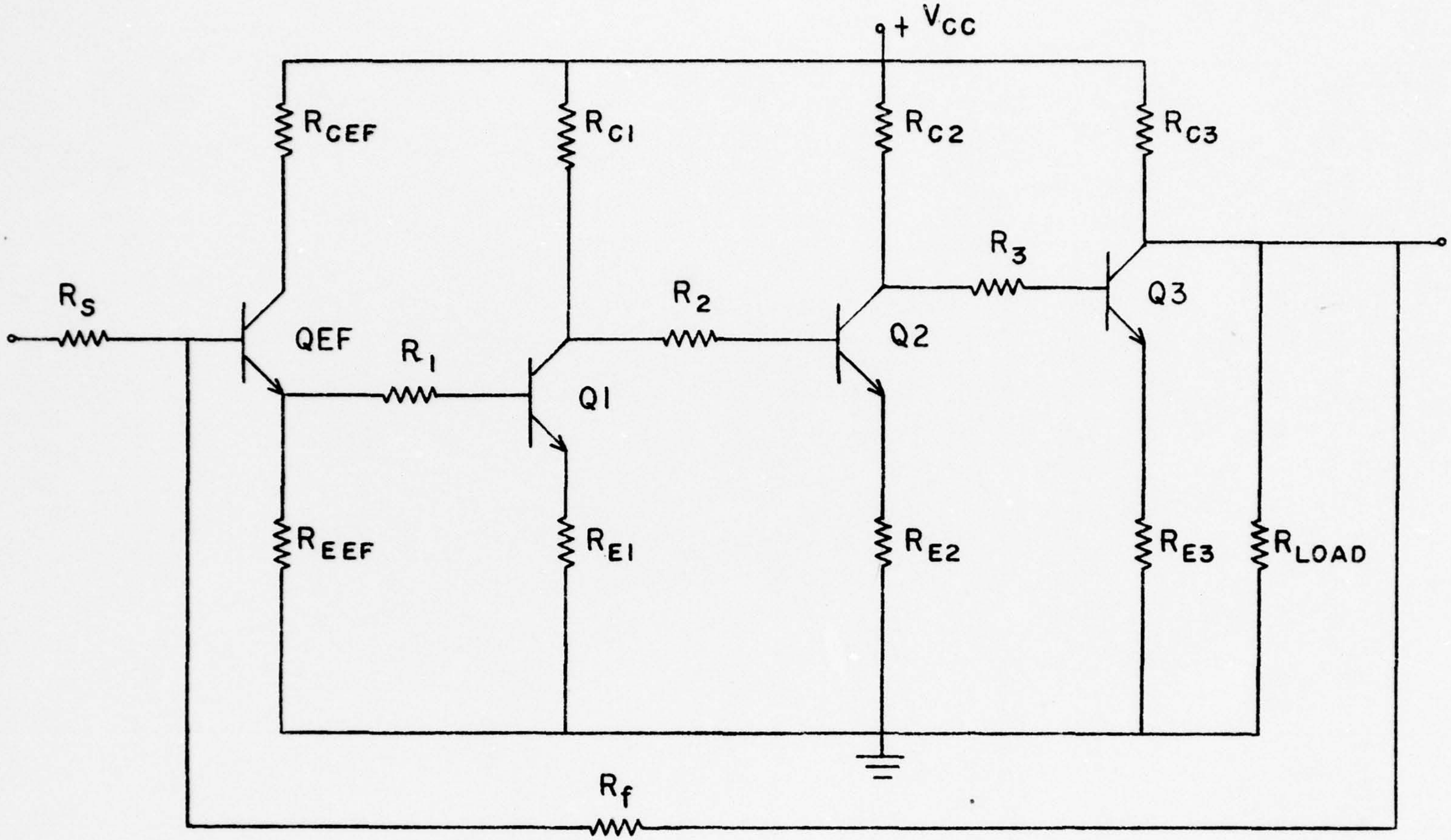

Figure 19. Circuit Diagram For Use With Program "Filter"

```
C      PROGRAM FILTER
C
C PROGRAM TO FIND THE  VALUE OF THE BASE RESISTANCE TO SET A
C PARTICULAR VALUE OF FO AND BW AND TO CALCULATE THE MIDGAIN.
C
       DIMENSION RE(3),R(3),RC(3),GM(3),RBE(3),RBB(3),CBE(3),CBC(3)
       DIMENSION CC(3),VERBE(3),ERBE(3),ER(3),EGM(3),VRC(3),TK(3)
       DIMENSION S(3)
C N IS THE NUMBER OF STAGES
C
C INITIALIZE
       FO=700E3
       BW=50E3
       RE(1)=200.
       RC(1)=2.E3
       GM(1)=40E-3
       RS=1.E3
       RLOAD=100.E6
       RBE(1)=3.75E3
       RBB(1)=100.
       CBE(1)=25E-12
       CBC(1)=3.E-12
C THETA IS THE EXCESS PHASE CORRECTION CONSTANT, K-THETA
       THETA=1.0
       N=3
C
C EMITTER FOLLOWER INITIAL VALUES
       REEF=1.E3
       HOCEF=0.0
       RBEEF=RBE(1)
       RBBEF=RBB(1)
       GMEF=GM(1)
       CBEEF=CBE(1)
```

```
      CBCEF=CBC(1)
      HFEEF=GMEF*RBEEF
      HIEEF=RBEEF+RBBEF
C
C SET ALL STAGES EQUAL TO FIRST STAGE
      DO 20 I=2,N
   20 RE(I)=RE(1)
      DO 21 I=2,N
   21 RC(I)=RC(1)
      DO 22 I=2,N
   22 GM(I)=GM(1)
      DO 23 I=2,N
   23 RBE(I)=RBE(1)
      DO 24 I=2,N
   24 CBE(I)=CBE(1)
      DO 25 I=2,N
   25 CBC(I)=CBC(1)
      DO 26 I=2,N
   26 RBB(I)=RBB(1)
C
C EMITTER FOLLOWER CALCULATIONS
      ROEF=1./(HOCEF+(1.+HFEEF)/(HIEEF+RS))
      RINEF=HIEEF+(1.+HFEEF)*REEF/(1.+HOCEF*REEF)
      YINEF=1./RINEF
      AVTT=1.-HIEEF/RINEF
C
C CALCULATE EFFECTIVE VALUES
      DO 10 I=1,N
   10 TK(I)=1./(1.+GM(I)*RE(I))
      DO 11 I=1,N
   11 EGM(I)=TK(I)*GM(I)
      DO 12 I=1,N
   12 ERBE(I)=RBE(I)/TK(I)
```

```
C
C CALCULATE WO,WC,AND ALPHA
      ALPHA=BW*3.14160
      WO=FO*6.28319
      WC=WO/1.73205+ALPHA
      DO 80 I=1,N
   80 S(I)=WC/THETA
C
C CALCULATE THE VALUE OF THE BASE RESISTORS
      YO=1./RC(N)+1./RLOAD
      R(N)=1./(S(N)*(TK(N)*CBE(N)+CBC(N)*(1.+EGM(N)*1./YO))-1.000/ERBE(N
     &))-RC(N-1)-RBB(N)
      I=N-1
   81 R(I)=1./(S(I)*(TK(I)*CBE(I)+CBC(I)*(1.+EGM(I)*RC(I)*(R(I+1)+RBB(I+
     &1)+ERBE(I+1))/(RC(I)+R(I+1)+RBB(I+1)+ERBE(I+1)))))-1.000/ERBE(I))-R
     &BB(I)-RC(I-1)
      I=I-1
      IF(I-1) 82,82,81
   82 R(I)=1./(S(I)*(TK(I)*CBE(I)+CBC(I)*(1.+EGM(I)*RC(I)*(R(I+1)+RBB(I+
     &1)+ERBE(I+1))/(RC(I)+R(I+1)+RBB(I+1)+ERBE(I+1)))))-1.000/ERBE(I))-R
     &BB(I)-ROEF
      DO 13 I=1,N
   13 ER(I)=R(I)+RBB(I)
C
C CALCULATE MID-BAND GAIN
      DO 14 I=1,N
      IF(I-1) 15,15,16
   15 VERBE(I)=ERBE(I)/(ER(I)+ERBE(I))
      GO TO 17
   16 VERBE(I)=VRC(I-1)*ERBE(I)/(ER(I)+ERBE(I))
   17 CC(I)=VERBE(I)*EGM(I)
      IF(I-N) 14,18,18
   14 VRC(I)=-CC(I)*RC(I)*(ER(I+1)+ERBE(I+1))/(RC(I)+ER(I+1)+ERBE(I+1))
```

```
   18 VOUT=-CC(N)*RC(N)*RLOAD/(RC(N)+RLOAD)
      GAIN=VOUT*AVTT
C
C CALCULATE G AND FB
      G=GAIN*WC*WC*WC
      FB=(-8.*(WC-ALPHA)**3)/G
      GF=FB*YO*(RS*YINEF+1.)/(RS*(YO-FB*(YINEF+YO)-FB/RS))
      RF=1./GF
C
C SCALE CAPACITANCES AND GM
      DO 99 I=1,N
      GM(I)=GM(I)/(1.E-3)
      CBC(I)=CBC(I)/(1.E-12)
   99 CBE(I)=CBE(I)/(1.E-12)
      GMEF=GMEF/(1.E-3)
      CBEEF=CBEEF/(1.E-12)
      CBCEF=CBCEF/(1.E-12)
C
C OUTPUT RESULTS
      WRITE(6,510) (I,I=1,N)
      WRITE(6,500) (R(I),I=1,N)
      WRITE(6,501) (RC(I),I=1,N)
      WRITE(6,502) (RE(I),I=1,N)
      WRITE(6,503) (GM(I),I=1,N)
      WRITE(6,504) (RBE(I),I=1,N)
      WRITE(6,505) (RBB(I),I=1,N)
      WRITE(6,506) (CBE(I),I=1,N)
      WRITE(6,507) (CBC(I),I=1,N)
      WRITE(6,602) THETA
      WRITE(6,508) RS
      WRITE(6,509) RLOAD
      WRITE(6,604)REEF,RBBEF,YINEF,RBEEF,ROEF,RINEF,GMEF,HOCEF,
     &CBEEF,CBCEF,AVTT
```

```
      WRITE(6,600) FO,BW
      WRITE(6,601) WO,WC,ALPHA
      WRITE(6,603) FB
      WRITE(6,100) GAIN
      WRITE(6,605) RF
      CALL EXIT
  100 FORMAT(' MIDBAND GAIN= ',E15.4)
  500 FORMAT(' R=',T9,3(F10.0,'  OHMS',4X))
  501 FORMAT(' RC=',T9,3(F10.0,'  OHMS',4X))
  502 FORMAT(' RE=',T9,3(F10.0,'  OHMS',4X))
  503 FORMAT(' GM=',T9,3(F10.0,' MMHOS',4X))
  504 FORMAT(' RBE=',T9,3(F10.0,'  OHMS',4X))
  505 FORMAT(' RBB=',T9,3(F10.0,'  OHMS',4X))
  506 FORMAT(' CBE=',T9,3(F10.2,'  PF  ',4X))
  507 FORMAT(' CBC=',T9,3(F10.2,'  PF  ',4X))
  508 FORMAT(/' R-LOAD=',F10.0,' OHMS')
  509 FORMAT(' R-LOAD=',F10.0,'  OHMS')
  510 FORMAT(T12,3('STAGE  ',I1,12X)/)
  600 FORMAT(//' FO=',E15.6,4X,'BW=',E15.6)
  601 FORMAT(' WO=',E15.6,4X,'WC=',E15.6,4X,'ALPHA=',E15.6)
  602 FORMAT(/' K-THETA=',F5.2)
  603 FORMAT('OK-FEEDBACK=',F15.6)
  604 FORMAT(//' EMITTER FOLLOWER STAGE:'/' RE=',T7,F7.0,' OHMS',T25,
     &'RBB=',T33,F7.0,' OHMS',T50,'YIN=',T57,E12.5,' MHOS'/' RBE=',T7,
     &F7.0,' OHMS',T25,'ROUT=',T33,F7.0,' OHMS',T50,'RIN=',T57,E12.5,
     &' OHMS'/' GM=',T7,F7.0,' MMHOS',T50,
     &'HOC=',T57,E12.5,' MHOS'/' CBE=',T7,F7.2,' PF',T25,'CBC=',T33,
     &F7.2,' PF',T50,'AVTT=',T57,E12.5)
  605 FORMAT(/' RF=',F7.0,' OHMS')
      END
```

```
                STAGE  1              STAGE  2              STAGE  3

R=            18022.   OHMS         16070.   OHMS         15185.   OHMS
RC=            2000.   OHMS          2000.   OHMS          2000.   OHMS
RE=             200.   OHMS           200.   OHMS           200.   OHMS
GM=              40.  MMHOS            40.  MMHOS            40.  MMHOS
RBE=           3750.   OHMS          3750.   OHMS          3750.   OHMS
RBB=            100.   OHMS           100.   OHMS           100.   OHMS
CBE=           25.00   PF            25.00   PF            25.00   PF
CBC=            3.00   PF             3.00   PF             3.00   PF

K-THETA= 1.00

R-LOAD=       1000. OHMS
R-LOAD=100000000.   OHMS


EMITTER FOLLOWER STAGE:
RF=     1000. OHMS        RBB=        100. OHMS      YIN=     0.64579E-05 MHOS
RBE=    3750. OHMS        ROUT=        32. OHMS      RIN=     0.15485E+06 OHMS
GM=       40. MMHOS                                  HOC=     0.0         MHOS
CBE=    25.00 PF          CBC=        3.00 PF        AVTT=    0.97514E+00


FO=    0.700000E+06     BW=    0.500000E+05
WO=    0.439823E+07     WC=    0.269640E+07     ALPHA=    0.157080E+06

K-FEEDBACK=         0.034883
MIDBAND GAIN=       -0.1915E+03

RF= 25490. OHMS
```

```
                STAGE  1                   STAGE  2                  STAGE  3

R=             10886.   OHMS             8925.   OHMS             8388.   OHMS
RC=             2000.   OHMS             2000.   OHMS             2000.   OHMS
RE=              200.   OHMS              200.   OHMS              200.   OHMS
GM=               40.  MMHOS               40.  MMHOS               40.  MMHOS
RBE=            3750.   OHMS             3750.   OHMS             3750.   OHMS
RBB=             100.   OHMS              100.   OHMS              100.   OHMS
CBE=            25.00   PF               25.00   PF               25.00   PF
CBC=             3.00   PF                3.00   PF                3.00   PF

K-THETA= 0.70

R-LOAD=       1000. OHMS
R-LOAD=100000000.   OHMS


EMITTER FOLLOWER STAGE:
RE=     1000. OHMS         RBB=        100. OHMS      YIN=     0.64579E-05 MHOS
RBE=    3750. OHMS         ROUT=        32. OHMS      RIN=     0.15485E+06 OHMS
GM=       40. MMHOS                                   HOC=     0.0         MHOS
CBE=    25.00 PF           CBC=       3.00 PF         AVTT=    0.97514E+00


FO=    0.700000E+06     BW=    0.500000E+05
WO=    0.439823E+07     WC=    0.269640E+07     ALPHA=    0.157080E+06

K-FEEDBACK=         0.022489
MIDBAND GAIN=      -0.2971E+03

RF= 41188. OHMS
```

APPENDIX C

MODIFICATION OF CUTOFF FREQUENCY DUE TO EXCESS PHASE SHIFT

Actual measurement of junction transistors show that at the cutoff frequency the phase shift of the common-emitter current gain, h_{fe}, is not 45^o, as predicted by the hybrid-π model, but is slightly larger than 45^o (26, pp. 296-303). This excess phase is directly dependent on the magnitude of the base built-in field, which is related to the steepness of the base impurity gradient. The results of this observed excess phase shift is to modify the single-pole roll-off characteristic of a single transistor amplifier stage used in (7) to

$$A(s) = \frac{A_o K_\theta \omega_c e^{\left[j \frac{K_\theta - 1}{\sqrt{K_\theta}} \frac{\omega}{\omega_\alpha}\right]}}{S + K_\theta \omega_c} \tag{C-1}$$

where ω_c is the common-emitter cutoff frequency, ω_α is the common-base cutoff frequency, and K_θ is the phase-correction constant. For a uniform base transistor, $K_\theta = 0.82$; for most graded-base transistors having error-function on gaussian-type impurity distributions, $K_\theta \approx 0.7 \pm 0.05$; for those graded-base transistors having steep exponential impurity distributions, $K_\theta \approx 0.6 \pm 0.05$. No excess phase is implied by $K_\theta = 1$. Since the frequencies of interest are those around ω_c, and since

$$\omega_c = \frac{1}{1+h_{fe}} \omega_\alpha \tag{C-2}$$

from which it is seen that $\omega_c << \omega_\alpha$, then the exponent term in (C-1) is very small and may be neglected to give

$$A(s) = \frac{A_o K_\theta \omega_c}{S+K_\theta \omega_c} \tag{C-3}$$

However, by comparison with (7), it is seen that the necessary open-loop pole location, ω_C, is

$$\omega_C + K_\theta \omega_c \tag{C-4}$$

or

$$\omega_c = \frac{\omega_C}{K_\theta} \tag{C-5}$$

Thus before the open-loop poles are set, the cutoff frequency must first be increased by a factor of $1/K_\theta$ to compensate for the reduction in the cutoff frequency due to the excess phase shift.